Inequality, Disaster Risk and the Great Recession

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Abstract

What drove large declines in aggregate quantities over the Great Recession? I study this question by building a dynamic stochastic overlapping generations economy where households hold both low-return liquid and high-return illiquid assets. In this economy, I explore how aggregate quantities and the distribution of households respond to a recession driven by an increased risk of a further economic downturn.

I find that a rise in disaster risk, and an empirically consistent fall in TFP, explain a significant fraction of the declines in aggregate consumption and investment observed over the Great Recession. Inequality is essential in driving these aggregate results. Comparing my model to an economy without illiquid assets, I show that household differences in both liquid and illiquid assets play a crucial role in amplifying the effects of a rise in disaster risk.

Keywords: Business cycles, incomplete markets, disaster risk, portfolio choice

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1 Introduction

During the Great Recession, the U.S. economy experienced sharp declines in aggregate quantities; the fall in consumption and investment, relative to trend, was around 4 and 18 percent, respectively.[1] These are unusually large declines in aggregate quantities given the relatively small drop in measured TFP, around 2 percent.[2,3] Furthermore, the micro data show that most households increased their savings rates during this recession.[4]

It has been a challenge for existing quantitative macroeconomic business cycle models to explain such large falls in aggregate quantities, in particular consumption, with the observed fall in TFP. For example, in their study of the Great Recession, Krueger et al. (2016) only explain a 2.5 percent drop in aggregate consumption, following a 4 percent drop in TFP.[5] In addition, they predict a counterfactual fall in households' savings rates in such a recession. Thus, in this paper, I try to provide a new quantitative framework that can reconcile macro- and micro-observations seen in the Great Recession. Specifically, in contrast to existing work, I explore aggregate dynamics when households fear a further economic downturn – economic disaster – while in a recession. In the face of such an increase in disaster risk, precautionary savings rise and consumption decreases. Importantly, this occurs whether or not there is an actual fall in income.

The assumed rise in the risk of an economic disaster, though parsimonious, is consistent with several salient features of the Great Recession. First, using the Michigan Survey of Consumers, De Nardi et al. (2012) find a significant fall in expected income growth for households during the Great Recession. Consistent with their empirical findings, in my model economy, a rise in the probability of economic disaster decreases households' expected future income. Actual economic disasters in the model involve sharp falls in wages, lower

[1]These are percentage point deviations from 2007Q4 levels. I have calculated these following the approach in Khan and Thomas (2013) with the capital and labor shares in my model economy. See data appendix A for more details.

[2]A relatively small drop in TFP in the 2008 recession is also documented in Kehoe et al. (2018) by comparing the two postwar recessions in the U.S. - 1982 recession and the Great Recession.

[3]Khan and Thomas (2013) and Gourio (2012) also pointed out that, during the financial crisis, there was no significant change in TFP.

[4]The rise in savings rates across households is documented in section 6.4.

[5]Similarly, in Guerrieri and Lorenzoni (2017), consumption falls only by 1 percent following a 10 percentage points drop in debt-to-GDP ratio.

returns on assets, and higher unemployment risk. When households rationally view the probability of such a deep recession to have risen, their expected future income falls, and this affects current consumption and savings decisions. Second, and more importantly, estimating the distribution of aggregate shocks to the economy using capital returns in 2007 and 2009, Kozlowski et al. (2017) indeed find that there is a significant rise in left tail aggregate risk in 2009, compared to the pre-crisis (2007) distribution. Consistent with their findings, such an increase in left tail risk is reproduced by a rise in disaster risk in my model economy. However, it is important to note that an increased probability of a disaster, on its own, fails to explain sharp declines in aggregate quantities. As I will show later, an important channel to amplify such an aggregate shock is households' differences in their holdings of both liquid and illiquid assets.

I study the Great Recession in a quantitative dynamic stochastic general equilibrium overlapping-generations (DSGE OLG) framework in which households' asset holdings vary with respect to returns and liquidity. An overlapping-generations framework is essential to explain the realistic consumption-savings behavior of households as infinite-lived agent models generate counter-factually high expenditure and low savings rates. In this framework, I explore how aggregate quantities and the distribution of households, over asset holdings, respond to a recession driven by an increased risk of a further economic downturn and a persistent negative TFP shock.

The model economy has three important features. First, households not only make consumption-savings decisions but also choose how to allocate savings across two types of assets – low-return liquid assets and high-return illiquid assets. As in Kaplan and Violante (2014), households have to pay fixed transaction costs to adjust illiquid wealth. These fixed costs determine a region of inaction where illiquid assets are not adjusted. Furthermore, costly portfolio adjustment implies liquidity risk in wealth, and households with different portfolios have different abilities to smooth consumption. Second, the aggregate economy faces a time-varying aggregate risk of economic disaster. Economic disasters are modeled as catastrophic declines in TFP, which lead to sharp falls in both wages and the price of illiquid assets, as well as a large unemployment risk. I assume that the probability of such a disaster sharply increased during the Great Recession.[6] Lastly, markets are incomplete; households face both uninsurable idiosyncratic earnings and unemployment risk. Importantly, unemployment risk, an important force for precautionary savings, rises in a recession as in Krusell and Smith (1998) and Krueger et al. (2016).

The calibrated economy reproduces much of the distributions of net worth, liquid assets, and illiquid assets seen in the 2007 SCF data. Moreover, while not targeted, fixed transaction costs for illiquid assets lead to model predictions for the share of illiquid assets, as a fraction of total assets, across the distribution of households, over wealth and age, that resembles the data. The model's disaster shock process is also consistent with the frequency and severity of disasters reported in Barro (2006).

Following a rise in disaster risk, and an empirically consistent fall in TFP, the benchmark model successfully predicts sharp falls in aggregate quantities over the Great Recession. In particular, it explains a 3.5 percent decline in aggregate consumption and a 15 percent drop in investment while, in the data, the corresponding declines are 3.6 percent and 18 percent, respectively. A heightened risk of disaster leads to a large negative wealth effect, which operates differently across households varying in age and wealth. For young and wealth-poor households, whose primary income source is labor earnings, a higher probability of a sharp fall in future wages and higher unemployment risk during an economic disaster decrease their expected future income. For old and wealthy households, who hold more illiquid assets, a fall in the expected future return on their savings decreases their expected future income.[7] Overall, an increased probability of lower expected future income gives rise to a larger fall in current consumption and an increase in precautionary savings. Moreover, as illiquid assets are costly to adjust, households increase their precautionary savings in safe liquid assets, which allow for better consumption smoothing.

In the model, illiquid assets correspond to physical capital in firms. A rapid drop in

[6]Disaster risk differs from an ordinary TFP shock. A rise in disaster risk leads to a higher probability of a large drop in TFP in the future, instead of an actual fall in current TFP. Moreover, in contrast to unemployment shocks or uncertainty shocks to individual earnings, the rise in disaster risk also captures the fall in the expected return on illiquid assets and thus affects the expected future income of households who hold illiquid savings.

[7]Economic disasters involve a sharp fall in TFP, alongside a rise in unemployment. As illiquid assets are held as capital in the model economy, a drop in TFP results in a fall in the return on illiquid assets.

investment results from the portfolio adjustment of some wealthy households toward safe, liquid assets. While many households do not change their holdings of illiquid assets, wealthy hand-to-mouth households, saving most of their wealth in illiquid assets, are more likely to liquidate part of their illiquid assets to smooth consumption in a recession. This amplifies the reduction of investment in physical capital.

Importantly, I find that allowing household differences in their holdings of both liquid and illiquid assets is crucial for explaining sharp falls in aggregate quantities following heightened disaster risk. In an otherwise comparable Aiyagari economy without illiquid assets, a rise in disaster risk and an empirically consistent fall in TFP only explain a 2.5 percent drop in aggregate consumption and a 8.8 percent decline in investment. Moreover, in this standard Aiyagari (1994) economy with a *single liquid asset*, held as physical capital, an increase in disaster risk changes aggregate dynamics little relative to the recession driven by a shock to current TFP alone.

In this single asset economy, a rise in precautionary savings following increased disaster risk must translate into investment in capital. This prevents the model from explaining the observed sharp fall in investment. It also reduces the equilibrium return to capital, and thus savings. Such a fall in the return to savings largely dampens households' desire to smooth consumption. Thus, the negative wealth effect from a heightened disaster risk is substantially offset by a large substitution effect.

In contrast, in my model economy with illiquid assets, though the expected return to illiquid assets falls with a rise in disaster risk, transaction costs weaken the resulting standard substitution effect. Specifically, more than 80 percent of households do not adjust their holdings of illiquid assets which is consistent with them being less responsive to a fall in the return on their savings. This sharply decreases the magnitude of the substitution effect compared to a model without illiquid assets.

Finally, to explore how well the model predicts households' behavior over the Great recession, I evaluate the model's consistency with the changes in household income, consumption, and expenditure rates seen in the PSID. Specifically, I examine the change in the annualized growth rates of disposable income and consumption, across wealth quintiles, between normal and recession times. I also study the percentage point change in expenditure rates. First, my model economy is consistent with a slowdown in the growth of disposable income and consumption, across all wealth quintiles, during the Great Recession. Second, similar to expenditure rate changes observed in the PSID over this period, a significant fraction of households increase their savings rates in a model recession accompanied by an increased risk of further economic downturn. Higher disaster risk leads to strong precautionary savings across households, allowing the model to better explain the rise in savings rates for the Great Recession than other existing business cycle models such as Krueger et al. (2016).[8] Lastly, I show that, though they hold a negligible share of total wealth in the economy, wealth poor households still play an insignificant role in determining the change in aggregate consumption during the recession.

The numerical method developed to solve the model may be of independent interest. First, I develop a two-stage approach to solve decision rules which involve a portfolio choice. This two-stage approach defines an intermediate value function over cash-on-hand, the choice of an illiquid asset, and idiosyncratic types. Next, I extend the backward Induction method of Reiter (2002, 2010) to solve a stochastic life-cycle model with a bivariate cross-sectional distribution of assets.[9,10] Compared to other existing methods to solve dynamic stochastic general equilibrium with heterogeneous agents, this method has advantages in terms of two things. First, this method does not linearize the model and thus does not rely on *certainty equivalence* in terms of aggregate shocks, as in Ahn et al. (2018). This allows the model to have a direct role for aggregate risk in household decisions and their distribution. Second, the backward induction method is robust when there is a large shock as it allows the distribution of households to vary in rich ways. This is especially important

[8]There may be other aggregate shocks that could deliver a rise in savings rates in the Great recession, such as an increase in unemployment risk or investment risk. Instead of claiming that disaster risk is the only way to forward, I suggest it as one source of risk to make the model consistent with important changes in expenditure rates seen in the micro data.

[9]Reiter (2010) also solves heterogeneous-agent models with aggregate shocks using the perturbation method. The idea of this method is to perturb the steady-state distribution with respect to aggregate shocks. However, as pointed out by Reiter (2010), this method is suitable for the model with small shocks as it linearizes the model.

[10]As pointed out in Kaplan and Violante (2014), solving for a stochastic OLG economy with two assets is challenging using the Krusell-Smith algorithm as this approach involves repeated long simulations to obtain an accurate parametric law of motion for the aggregate state. This simulation step is exceptionally costly for my model economy as equilibrium prices are not determined by the beginning-of-period state of the economy.

in a model with multiple assets when many households show large changes in their asset holdings following aggregate shocks.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 presents the model economy. Section 4 discusses the calibration and Section 5 discusses numerical methods. Section 6 presents the quantitative results for aggregate and household dynamics. Section 7 concludes.

2 Related literature

This paper contributes to the literature that explores household-level heterogeneity and its aggregate implications for the business cycle. In their seminal paper, Krusell and Smith (1998) found that the first moment of the distribution of wealth was sufficient to describe the dynamics of aggregate variables. However, in recent work, Krueger et al. (2016) find that the distribution of wealth plays a significant role in explaining changes in aggregate consumption. Specifically, they show that a model economy with a more pronounced dispersion in wealth experiences a larger drop in aggregate consumption compared to the original Krusell-Smith economy with less inequality. However, in the absence of a portfolio choice for households, the larger fall in consumption, in their model economy, is accompanied by a smaller fall in investment. Moreover, as mentioned before, their model is not able to explain the observed rise in savings rates during the Great Recession.

Guerrieri and Lorenzoni (2017) study the effects of a credit crunch on consumer spending in a heterogeneous-agent incomplete markets model.[11] They find that the credit crunch forces financially constrained households to repay their debts and increases precautionary savings by unconstrained households. This pushes down the equilibrium real interest rate. Financially unconstrained households respond to this fall in the real interest rate by increasing their consumption. This counteracts the direct effect of a credit shock in their model, generating a modest drop in aggregate output and consumption.[12] Even in their

[11]Solving the model economy without capital, their model is silent on the fall in investment in the Great Recession.

[12]In their baseline economy without nominal rigidities, a 10 percentage point drop in a debt-to-GDP ratio only leads to a one percent drop in output and consumption.

extended model with durable goods, consumption and output increase by 0.4 percent, following a credit crunch, due to the high substitutability of bonds and durables and the high interest-elasticity of durable purchases.[13,14]

Despite the important advances made in these papers, existing quantitative business cycle theories find it challenging to explain a sharp fall in aggregate consumption during the Great Recession, in addition to the fall in investment, without relying on a counterfactually large drop in TFP or a demand externality.[15] My work contributes to this literature by introducing two additional channels. First, I introduce time-varying aggregate risk, which alters households' expected future income and the return on risky assets. Second, I allow differences in households not just in the level of wealth but also in their holdings of liquid and illiquid assets. I find that this additional dimension of heterogeneity has a vital role in amplifying aggregate risk and understanding the sharp declines in consumption and investment over the 2008 recession.

My focus on household portfolio choice is in the same spirit as Kaplan and Violante (2014), who study consumption responses to fiscal stimulus in an incomplete-markets life-cycle model with liquid and illiquid assets. Introducing nominal rigidities, Kaplan et al. (2018) also study the transmission mechanism of monetary policy to households' consumption in this setting. By contrast, I focus on how this endogenous portfolio channel amplifies the effect of a rise in disaster risk in a recession. As mentioned before, such a rise in left tail aggregate risk during the Great Recession is well documented in Kozlowski et al. (2017), who study its importance for the responses of aggregates. However, their emphasis lies on firm dynamics while my paper emphasizes household dynamics.

Gourio (2012) first introduced disaster risk into a real business cycle model, studying its macroeconomic and asset pricing implications in a representative agent model.[16] In this

[13] A fall in the interest rate on bonds makes households adjust their portfolios from bonds to durable purchases, increasing output.

[14] The relatively high substitutability of two assets in Guerrieri and Lorenzoni (2017) is driven by the following facts. First, they model illiquidity in durables by introducing a discount on resale values instead of adjustment costs. Second, both durables and non-durables are produced using the same technology, making the purchase prices of the two goods identical. In contrast, the fixed adjustment costs embedded in illiquid wealth in my model economy weakens the substitutability between the two assets. Also, illiquid assets are productive capital subject to aggregate risk in my model economy.

[15] See Krueger et al. (2016) for the role of demand externality.

[16] There is no portfolio choice in his model.

setting, he explains the fall in investment in response to a rise in disaster risk. However, this fall in investment coincides with a counter-factual increase in consumption. By contrast, in my model economy, a heightened risk of disaster decreases both aggregate consumption and investment. Wealth-poor households increase their precautionary savings in response to a rise in aggregate risk, thus playing a significant role in explaining the consumption response. The response in investment is mainly driven by wealthy households who initially hold more of their wealth in high-yield illiquid assets. This emphasizes the importance of household heterogeneity in shaping aggregate responses following a rise in disaster risk.[17]

The disaster shock in my model economy is related to uncertainty shocks, news shocks or financial shocks that may change households' expectations of future income. Bayer et al. (2019) study the macroeconomic effects of a rise in uncertainty in household income in a heterogeneous-agent New-Keynesian model, explaining monetary and fiscal stabilization policy in such a setting. In contrast to individual-level income risk, my paper focuses on a rise in aggregate risk, which affects not just labor earnings but also the return to capital. More importantly, disaster risk affects both the first and second moments of the aggregate shock introduced here. The former is essential in delivering a sharp decrease in expected future income across households. This helps the model explain a more persistent and severe recession compared to a pure rise in the uncertainty in household earnings.[18] Jaimovich and Rebelo (2009) study the role of news shocks for the business cycles. However, they abstract from household heterogeneity that turns out to be a crucial amplification channel in my model economy. They also study the implications of news shocks in a perfect foresight setting, with no role for aggregate risk.

My emphasis on the distributional effects of the recession across households with different asset portfolios is similar to Glover et al. (2019) and Hur (2018). Glover et al. (2019) study the intergenerational redistribution effects of the Great Recession in a stochastic complete-markets OLG endowment economy. Hur (2018) also examines the distributional

[17]See online appendix B for the representative agent model with liquid and illiquid assets and disaster risk.

[18]Note also that the mechanics in New-Keynesian models are different from those in standard real business cycle models as they rely heavily on nominal rigidities, which induce large demand effects. For example, as shown in Figure 19 in Bayer et al. (2019), in the absence of price stickiness, a rise in idiosyncratic income risk leads to an investment boom and little change in output.

consequences of the Great Recession on different generations by extending their model to an incomplete-markets economy with borrowing constraints. Studying financial markets characterized by two assets, a risk-free bond and risky equity, both papers show significant disparities in welfare losses across different generations. For example, Glover et al. (2019) find that older cohorts are hurt more than younger cohorts when there is a larger decline in asset prices than that in wages. Hur (2018) finds that young risky asset holders suffer the largest welfare losses during the 2008 recession. These models are calibrated to match the price dynamics in the Great Recession, which are important in their welfare results. Instead of focusing on the welfare implications of the 2008 recession, which have been thoroughly explained in these studies, my work develops a DSGE OLG framework that can explain the role of both aggregate and individual risk in determining large changes in aggregate quantities in a production economy.

3 Model

The model economy consists of four types of agents: households, a perfectly competitive investment firm, a production firm, and a government. Households differ by age, wealth, productivity, employment status, and in their subjective discount factors. Each period, a household chooses its consumption, total savings, and portfolio of liquid and illiquid assets. As in Kaplan and Violante (2014), a household has to pay a portfolio adjustment cost if it chooses to adjust its illiquid assets and re-balance its portfolio. Illiquid assets are capital held by the investment firm and rented to the production firm. The government supplies the net quantity of liquid assets, which are government debt. The definition of recursive competitive equilibrium is described in Appendix C.

3.1 Households

This is a life-cycle model where households live for a finite number of periods. Using j to index their years of life, they start working at $j = 1$, and retire at J_r, and their last period is J. Note that households have permanent differences in time discount factors

$\beta \in \{\beta_L, \beta_M, \beta_H\}$ which are fixed over their lifetime.[19]

During the working lives, households face stochastic idiosyncratic unemployment, $e \in \{0, l_e, 1\}$, which determines their working time. As in Krusell and Smith (1998) and Krueger et al. (2016), this unemployment risk evolves as a function of the exogenous aggregate state, z. Here, z is a vector summarizing the exogenous aggregate state. This captures the rise in unemployment in recessions and allows aggregate hours worked to varying cyclically.

Specifically, households can be completely unemployed ($e = 0$) with probability $\pi_u(z)$, partially employed ($e = l_e$) with $\pi_l(z)$, or employed as full-time workers ($e = 1$) with the remaining probability. I introduce partial (un)employment to match the mean and median unemployment durations, which are less than a model period of one year following Khan and Lidofsky (2019). Partially and fully unemployed workers receive unemployment benefits from the government proportional to their lost earnings and unemployment duration. The replacement rate is θ_u.

Households also draw idiosyncratic productivity shocks ε during their working lives. These productivity shocks follow a Markov chain $\varepsilon \in \{\varepsilon_1, ..., \varepsilon_{n_\varepsilon}\}$, where $Pr(\varepsilon' = \varepsilon_k | \varepsilon = \varepsilon_l) = \pi_{lk} \geq 0$ and $\sum_{k=1}^{n_\varepsilon} \pi_{lk} = 1$. After retirement, households receive lump-sum social security benefits proportional to their last earnings shock, $s(\epsilon^{Jr-1}) = \theta_s w(z, \mu) \epsilon^{Jr-1}$.[20] Here, $w(z, \mu)$ is the hourly wage.

The economy has two assets: high-yield illiquid assets, a, and low-yield liquid assets, b. Each unit of the illiquid asset pays a dividend $d(z, \mu)$ and has an ex-dividend price $p(z, \mu)$. Adjusting illiquid assets to a value other than their non-adjusted, after-tax, post-dividend balance involves a fixed adjustment cost, $\xi(\kappa, a)$.[21] This cost is in units of output, and proportional to the share of illiquid wealth to net worth, $\kappa = \frac{a}{a+b}$. The functional form for

[19] Preference heterogeneity is introduced to match the observed distribution of wealth as in Krusell and Smith (1998) and Krueger et al. (2016).

[20] Social security benefits are paid based on the average of the highest 35 years of earning by the Social Security Administration (SSA). In the model, calculating average earnings requires one more state variable, making computation more challenging. Given the high persistence of earnings, I proxy the history of earnings over a worker's life-cycle using the level of earnings in the last working period.

[21] This adjustment cost is fixed in terms of the choice of illiquid assets for the next period.

the fixed adjustment cost is

$$\xi(\kappa, a) = \xi_0 + \xi_1 |\kappa| a.^{22}$$

Households paying these costs adjust illiquid wealth to a desired value; otherwise, the after-tax dividend payments and principal are re-invested in illiquid assets.[23] Borrowing is only allowed in liquid assets and positive liquid asset pays a return $\frac{1}{q(z,\mu)}$.

In sum, each period, a household is identified by its age $j \in \mathbf{J} = \{1, ..., J\}$, illiquid asset, $a \in \mathbf{A} \subset \mathbf{R}_+$, liquid asset, $b \in \mathbf{B} \subset \mathbf{R}$, working status $e \in \{0, l_e, 1\}$, productivity $\varepsilon \in \mathbf{E}$, and time discount factor $\beta \in \{\beta_L, \beta_M, \beta_H\}$. The distribution of households, μ, is defined over $(j, a, b, e, \varepsilon, \beta)$ and evolves following the mapping $\mu' = \Gamma(z, \mu)$.

Discrete portfolio adjustment choice. After any portfolio adjustment decision, a household realizes the value $v_j(a, b, e, \varepsilon_l, \beta; z_f, \mu)$ given the aggregate state (z_f, μ).

$$v_j(a, b, e, \varepsilon_l, \beta; z_f, \mu) = \max \left\{ v_j^a(a, b, e, \varepsilon_l, \beta; z_f, \mu), v_j^n(a, b, e, \varepsilon_l, \beta; z_f, \mu) \right\} \tag{1}$$

Here, v_j^a represents the value of a household adjusting its illiquid assets and v_j^n is the value of a non-adjusting household.

[22] The aggregate results are robust to excluding the second component of the adjustment cost which is paid proportional to the current holdings of illiquid assets. This term is introduced to better match the highly skewed distribution of liquid assets in data. With proportional costs, rich households hold a relatively large amount of liquid assets to finance their portfolio adjustment costs.

[23] Compared to resource costs, utility costs of adjusting illiquid assets drive different portfolio adjustment behavior in households. Specifically, given the relatively low marginal utility of rich households, utility costs lead to a lower probability for the rich of adjusting their illiquid assets, compared to that for the poor. By contrast, with resource costs, rich households can better afford such costs and hence have a higher probability of adjusting their illiquid assets than the poor.

Following Kaplan and Violante (2014), I assume Epstein-Zin preferences.[24] Let

$$v_j^0(a, b, \varepsilon_l, \beta; z_f, \mu) = \left\{ \sum_{k=1}^{n_\varepsilon} \pi_{lk} \sum_{g=1}^{n_z} \pi_{fg}^z \sum_{e \in \{0, l_e, 1\}} \pi_e(z_g) v_j(a, b, e, \varepsilon_k, \beta; z_g, \mu)^{1-\gamma} \right\}^{\frac{1}{1-\gamma}}$$

be the expected value of a household at the beginning of the period before any shock is realized. This is the Epstein-Zin certainty equivalent function with $\sigma > 0$ and the coefficient of relative risk aversion, $\gamma > 0$. Note that the Epstein-Zin preference allows the elasticity of intertemporal substitution (IES) to differ from the inverse of the coefficient relative risk aversion.

A household that adjusts illiquid asset holdings. I describe the optimization problem of a household that adjusts its illiquid asset holdings.

$$v_j^a(a, b, e, \varepsilon_l, \beta; z_f, \mu) = \max_{c,\, a',\, b'} \left[(1-\beta) c^{1-\sigma} + \beta v_{j+1}^0(a', b', \varepsilon_l, \beta; z_f, \mu')^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{2}$$

$$\text{s.t. } c + p(z_f, \mu) a' + q(z_f, \mu) b' \leq (p(z_f, \mu) + (1-\tau_a) d(z_f, \mu)) a + b + x - \xi(\kappa, a)$$

$$x = \begin{cases} (1-\tau_n) w(z_f, \mu) \varepsilon_l (e + (1-e)\theta_u) & \text{if } j < J_r \\ (1-\tau_n) s(\epsilon^{Jr-1}) & \text{otherwise} \end{cases}$$

$$b' \geq \underline{b}, \ a' \geq 0, \ c \geq 0$$

$$\mu' = \Gamma(z_f, \mu)$$

where $\underline{b}$ is the borrowing limit. Here, x is labor income before retirement and social security income after retirement. A worker, $e = l_e$ or 1, receives labor income and an unemployed worker, $e = 0$, receives unemployment benefits. A retiree, $j \geq J_r$, receives a social security benefit, $s(\epsilon^{Jr-1})$. Labor income is taxed at τ_n and dividend income is taxed at τ_a.

[24] In online Appendix C, by solving a model with CRRA preference, I show that non-separable preferences play an important role in reproducing empirically consistent distribution of liquid assets and the fraction of wealthy hand-to-mouth households. I also show that, once calibrated, the aggregate results are robust in a model with standard CRRA preference.

A household that does not adjust illiquid asset holdings. If a household does not pay its fixed cost, it can only choose its consumption and stock of safe liquid wealth for the next period, b', as described below. Illiquid assets pay after-tax dividends, which are re-invested into illiquid assets with the principal.

$$v_j^n(a, b, e, \varepsilon_l, \beta; z_f, \mu) = \max_{c,\, b'} \left[(1-\beta)c^{1-\sigma} + \beta v_{j+1}^0(a', b', \varepsilon_l, \beta; z_f, \mu')^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{3}$$

$$\text{s.t. } c + q(z_f, \mu)b' \leq b + x$$

$$x = \begin{cases} (1-\tau_n)w(z_f, \mu)\varepsilon_l(e + (1-e)\theta_u) & \text{if } j < J_r \\ (1-\tau_n)s(\epsilon^{Jr-1}) & \text{otherwise} \end{cases}$$

$$p(z_f, \mu)a' = (p(z_f, \mu) + (1-\tau_a)d(z_f, \mu))a$$

$$b' \geq \underline{b},\ c \geq 0$$

$$\mu' = \Gamma(z_f, \mu)$$

3.2 Competitive investment firm

A competitive investment firm owns the technology that creates capital and rents its capital to a production firm at a rental rate $r^k(z, \mu)$. The investment firm sells shares of total capital at an ex-dividend price $p(z, \mu)$ and pays dividends $d(z, \mu)$ to households. It also sells shares of future capital, $p(z, \mu)k'$, to households. The revenue from these shares is used to transform k' units of current output into capital for the next period. Moreover, the investment firm faces a convex capital adjustment cost.

$$\Phi(k', k) = \left(\frac{k' - k}{k}\right)^2 k$$

This quadratic adjustment cost allows the price of capital to deviate from that of consumption as it makes consumption and capital less than perfectly substitutable. This helps the model generate a fall in the price of illiquid assets in a recession.

Below, I explicitly describe the optimization problem of the investment firm.

$$J(k;z_f,\mu) = \max_{k'}\Bigg((r^k(z_f,\mu)+1-\delta)k - (p(z_f,\mu)+d(z_f,\mu))k \\ + p(z_f,\mu)k' - k' - \Phi(k',k) + \sum_{g=1}^{n_z}\pi^z_{fg} r(z_g,z_f,\mu)J(k';z_g,\mu')\Bigg)$$

where investment firm discounts future earnings by $r(z_g,z_f,\mu)$.[25,26]

3.3 Production firm

The production firm employs capital k and hires labor n to produce output through a CRS production function $y = f(z)k^\alpha n^{1-\alpha}$, where $0 < \alpha < 1$ and $f(z) > 0$. Thus, the optimization problem of the production firm is

$$\max_{k,n}\big(f(z)k^\alpha n^{1-\alpha} - r^k(z,\mu)k - w(z,\mu)n\big)$$

Here, $f(z)$ can take five different values depending on the aggregate state of the economy. This involves a disaster, a recession with a heightened disaster risk, an ordinary recession without disaster risk, normal times, and boom.

3.4 Government

The government supplies liquid assets B_s at a price $q(z,\mu)$. Social security benefits and unemployment benefits are also taxed at τ_n. Government revenues are used to finance social security benefit payments, unemployment benefits payments, interest payments on

[25] In equilibrium, this discount factor should be equal to the marginal rate of substitution of households who hold positive illiquid assets. In the model economy, it is hard to determine the MRS as heterogenous households decide their portfolio choices endogenously. In Appendix B, I show how the equilibrium prices and dividend payments to the shares of capital are determined. As the investment firm is assumed to earn zero profits in every period, the equilibrium prices are not affected by this discount factor.

[26] A caveat on standard real business cycle theories is that they fail to explain the empirical volatility of asset prices (see Rouwenhorst (1995), Danthine et al. (1992) and Tallarini Jr (2000)). Unfortunately, quadratic adjustment cost in my model is not enough to address this issue, only generating a 2 percent drop in the equilibrium price of capital with a rise in disaster risk. This also makes my framework less suitable for the welfare analysis compared to those in Glover et al. (2019) and Hur (2018).

debt, and government spending $G_s(z, \mu) \geq 0$. Every period, the government budget is balanced.[27]

4 Calibration

In this section, I describe how I calibrate the model economy. Assuming a model period of one-year, households enter the labor market at age 25, retire at age 65, and live until age 84 with certainty. I first explain the parameters directly affecting the wealth distribution of households. Second, I discuss the aggregate shock processes. Lastly, I describe the calibration strategy for parameters governing unemployment and earnings shock processes.

4.1 Distribution of net worth, illiquid and liquid wealth

As summarized in Table 1, I calibrate six parameters $(\beta_L, \beta_M, \beta_H, \xi_0, \xi_1, \underline{b})$ for subjective discount factors, the adjustment cost function, and the borrowing limit, to match six moments of the wealth distribution from the 2007 SCF.

Table 1: Moments targeted

moments	data	model
capital to output ratio	2.66	2.66
share of liquid asset to output	0.35	0.35
wealth Gini	0.78	0.78
fraction of households holding illiquid wealth	0.73	0.73
fraction of households with positive illiquid wealth but no liquid wealth	0.32	0.20
fraction of households holding zero or negative net worth	0.103	0.16

Source: 2007 SCF

I first calibrate the model economy to match a capital to output ratio of 2.66. Here, capital is measured as productive illiquid wealth. Following Kaplan et al. (2018), productive illiquid wealth includes business equity, stocks and net equity in non-residential real estate

[27] Government spending is chosen as residual to balance the government budget constraint.

as well as 40 percent of net housing and net consumer durables.[28] All other remaining assets and debts are considered liquid. Given that a sampling unit in the SCF is a household, the total value of productive illiquid assets is divided by the average family size in 2007 to make it comparable to GDP per capita.[29] The calibrated economy gives rise to a 9.3 percent return on illiquid wealth in the steady-state when a 1 percent return on liquid wealth is targeted. This results in a liquidity premium of 8.3 percent.

The model economy is also calibrated to match the two types of liquidity constrained households; households with zero or negative net worth (poor hand-to-mouth households) and households with positive illiquid wealth but no liquid wealth (wealthy hand-to-mouth households).[30] Kaplan and Violante (2014) point out that these households have large consumption responses following transitory income changes.[31] Interestingly, though my model economy is calibrated to match the share of wealthy hand-to-mouth households, I find that the aggregate response of consumption is not very sensitive to this moment. This is because, in my recession, the fall in expected future income across households is persistent. Moreover, wealthy hand-to-mouth households can easily liquidate their illiquid wealth to smooth out their consumption when facing a large change in income. Thus, the importance of wealthy hand-to-mouth households is different when considering recessions that involve more persistent large falls in income than transitory income changes such as fiscal rebates in Kaplan and Violante (2014).

[28] Stocks are considered illiquid as most of them are held in retirement accounts or involve management fees. Following Glover et al. (2019), money market mutual funds and quasi-liquid retirement accounts are included in stocks. To be consistent with the model economy, which does not involve collateralized borrowing, I measure residential property net of all debt secured by residential property (mortgages, home equity loans, and HELOCs) in illiquid wealth as in Kaplan and Violante (2014). In using the net value of housing wealth, I abstract from the issuance of home equity loans, which provide liquidity. Introducing home equity loans in the model would provide an additional incentive to hold illiquid assets, making households somewhat less responsive to the fall in their expected return.

[29] As GDP per capita is expressed in chain-weighted 2009 dollars, I adjust the value of illiquid wealth to 2009 dollars using CPI-IPUMS.

[30] These liquidity constrained households are comparable to the poor and wealthy hand-to-mouth households as defined in Kaplan and Violante (2014) and Kaplan et al. (2018). Kaplan and Violante (2014) define wealthy hand-to-mouth households as those with liquid wealth less than half of their earnings but holding positive balances of illiquid wealth. They estimate a total fraction of hand-to-mouth households between 17.5 percent and 35 percent of households in the 2004 SCF, including both wealthy and poor households. Among hand-to-mouth households, they find 40 to 80 percent are wealthy. I instead define wealthy hand-to-mouth households as those with no liquid wealth but positive illiquid wealth.

[31] Kaplan and Violante (2014) showed that the average MPC for hand-to-mouth households is around 40 percent, while that for non-hand-to-mouth households is only 7 percent.

Importantly, the calibrated economy successfully reproduces most of the distributions of the net worth, illiquid, and liquid wealth seen in the data. Table 2 compares these distributions in the 2007 SCF to those in the steady-state of the benchmark economy with liquid and illiquid assets. In Table 2, I have only targeted the share of households with zero or negative net worth and the wealth Gini coefficient. Nonetheless, the model economy still reproduces much of the dispersion in wealth. As seen in the top panel, the quintile distribution of net worth in the benchmark economy is close to that in the 2007 SCF, explaining more than 81 percent of the total wealth held by the wealthiest 20 percent of households.[32]

Table 2: Distributions of net worth, illiquid assets and liquid assets

Net worth	Q1	Q2	Q3	Q4	Q5	≤ 0	Gini
2007 SCF	-0.3	1.4	5.7	14.1	79.1	10.3	0.78
Benchmark	-0.0	0.6	3.1	14.8	81.7	17.0	0.78
Single asset	0.0	0.7	3.5	16.2	79.6	17.0	0.75
Illiquid wealth	Q1	Q2	Q3	Q4	Q5		Gini
2007 SCF	0.14	1.6	5.9	14.4	78.0		0.76
Benchmark	0.0	0.5	3.0	14.2	82.4		0.77
Liquid wealth	Q1	Q2	Q3	Q4	Q5		Gini
2007 SCF	-11.7	-0.53	0.92	7.8	103		0.92
Benchmark	-0.2	-1.30	1.33	9.7	92		0.85

Notes: Table 2 shows the share of net worth, illiquid assets, and liquid assets across the wealth quintiles. It also reports the share of households with zero or negative net worth and the Gini coefficient for net worth, illiquid assets, and liquid assets in the 2007 SCF and model economies.

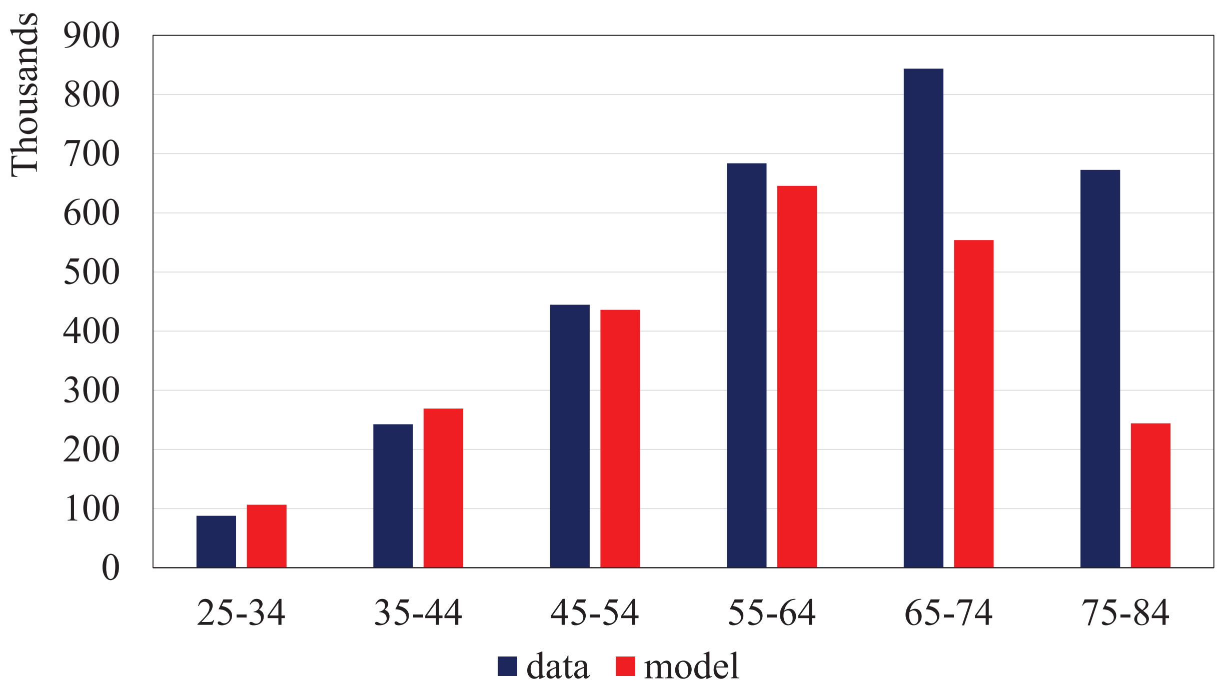
Table 2 also shows the model's success in explaining the distributions of liquid and illiquid wealth in the 2007 SCF data. The distributions of illiquid assets are comparable to those in the data. The model economy reproduces a Gini of 0.77 and shows 82 percent of all illiquid assets as being held by the wealthiest 20 percent of households, compared to 0.76 and 78 percent, respectively, in the data. The data shows more dispersion in liquid assets than that in illiquid assets; the liquid wealth Gini is 0.92 and borrowing by the bottom 20

[32]The resulting skewed distribution of wealth is not driven by a high fraction of households near the borrowing limit as in Huggett (1996) but by the right tail of the distribution. While not shown in Table 2, the wealthiest 10 percent of households hold 60 percent of the wealth in the model economy compared to 64 percent in the 2007 SCF.

percent of households represents approximately 12 percent of the total stock. Although the observed distribution of liquid assets is more skewed than that in the model, the benchmark economy still generates a greater dispersion in liquid assets relative to illiquid assets. This drives a higher liquid wealth Gini of 0.85.

Further untargeted consistency between model and household data is seen in wealth profiles over age, and illiquid wealth shares over wealth and age. While not targeted, the calibrated economy first gives rise to life-cycle wealth profiles over ages, similar to the data. Figure 1 compares the average levels of households' wealth in the data to those in the model over age groups. Though it predicts a relatively fast de-accumulation of wealth for the old compared to the data, the hump-shaped life-cycle profile of wealth in the model aligns well with that in the data.[33]

Figure 1: Household average wealth over age groups

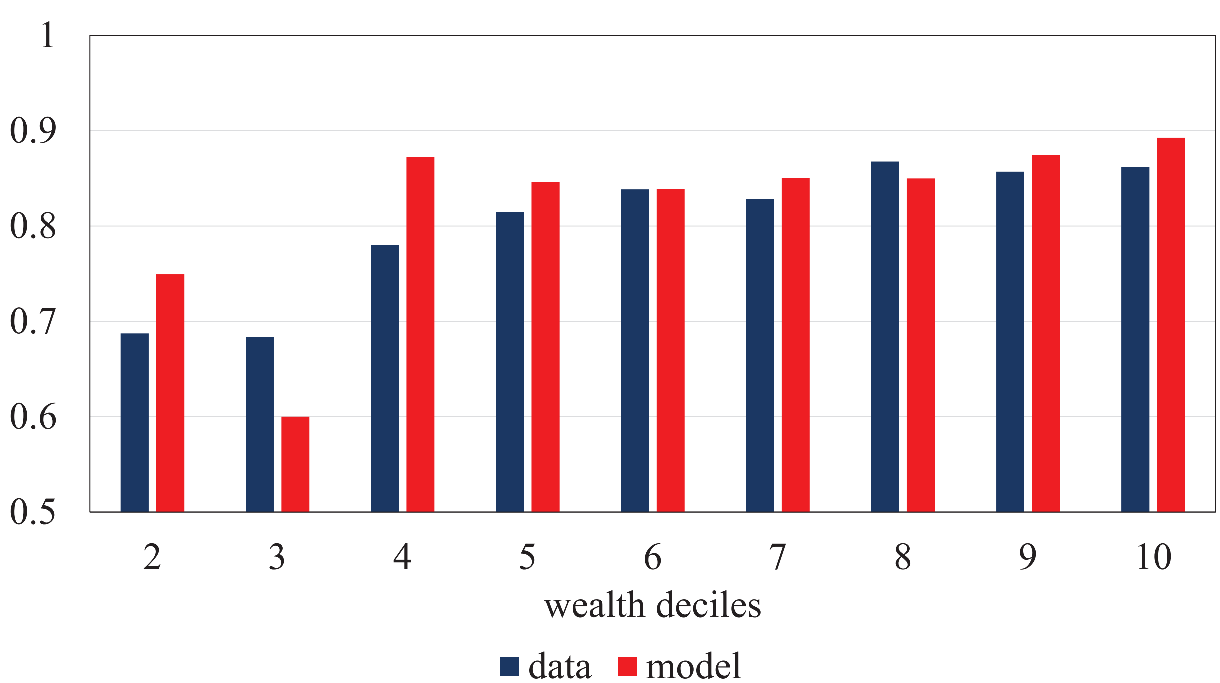


Notes: The household average level of wealth (in 2007 dollars) over age groups in the 2007 SCF (blue bars) and the benchmark economy (red bars).

[33]Without any mortality risk and bequest motive, it is hard to explain households' savings behavior after retirement. See De Nardi (2004) and De Nardi et al. (2016) for further discussion.

In Figure 2, I compare the shares of illiquid assets to total assets, in the model, to those in the 2007 SCF, over wealth deciles. I dropped the first wealth decile group as some households in this group have little total assets, making the ratio uninformative. Figure 2 shows that the benchmark economy produces shares that again capture patterns seen in the data. In general, wealthy households hold more of their savings in the illiquid asset as it pays a higher return and they can more easily afford adjustment costs.

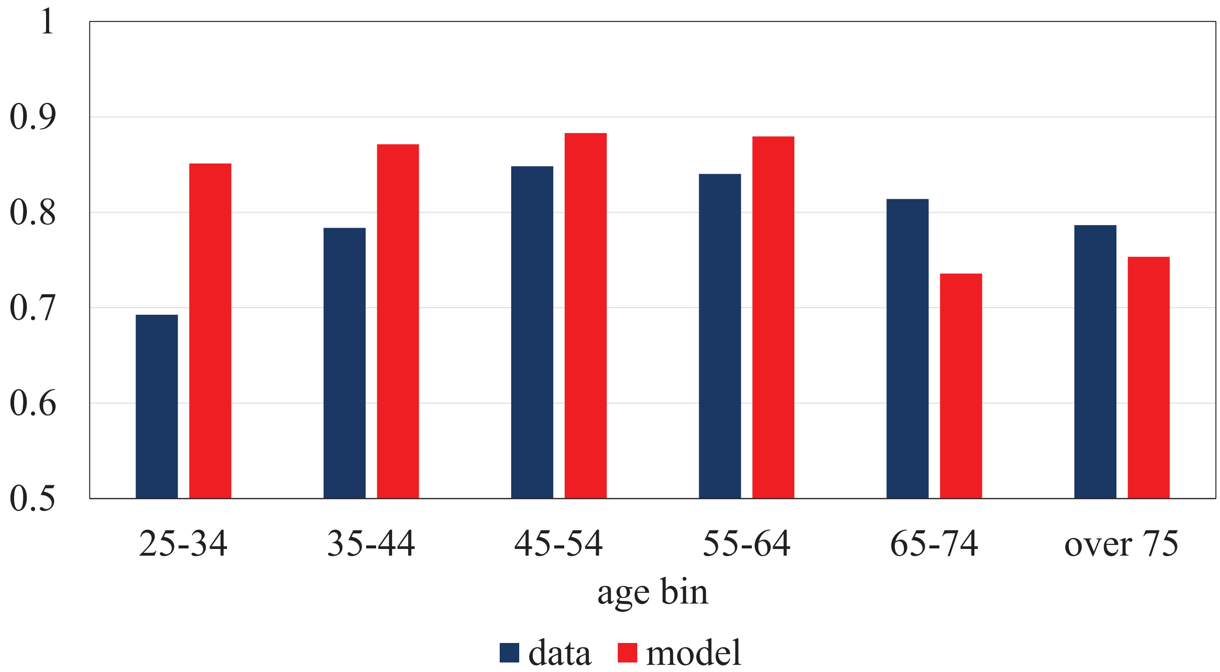
Figure 2: Share of illiquid assets as a fraction of total assets over wealth deciles



Notes: The average share of illiquid assets as a fraction of total assets over wealth deciles in the 2007 SCF (blue bars) and the benchmark economy (red bars).

Figure 3 shows the corresponding figure over different age groups. Over the first two groups, the benchmark economy initially overestimates the average share of illiquid assets. However, it captures the increase in this share until the 55-64 age group and its decline, thereafter. This is consistent with the fact that older cohorts tend to increase their savings in liquid assets because they have less time to recover from any fall in asset prices, as pointed out in Glover et al. (2019).

Figure 3: Share of illiquid assets as a fraction of total assets over age groups



Notes: The average share of illiquid assets as a fraction of total assets over age groups in the 2007 SCF (blue bars) and the benchmark economy (red bars).

4.2 Aggregate shocks: TFP and disaster shock

The exogenous aggregate state z consists of ordinary total factor productivity (TFP) shock η and a disaster state $\widetilde{d}$. Aggregate TFP is assumed to follow an AR(1) process in logs, $\log \eta' = \rho_\eta \log \eta + \varepsilon_\eta$, where $\varepsilon_\eta \sim N(0, \sigma_\eta^2)$. I estimate this process using Solow Residuals calculated from data on real U.S. GDP, private capital, and the total hours worked over 1954Q1 to 2012Q4. I discretize the estimated process into three values, $\eta \in \{\eta_1, \eta_2, \eta_3\}$ with a Markov chain $Pr(\eta'|\eta) = \pi^\eta \geq 0$ using Rouwenhorst's (1995) method.[34,35]

[34]Compared to Tauchen (1986), Rouwenhorst method can accurately approximate the continuous shock process with a smaller number of discretization points (see Kopecky and Suen (2010)). When I increase the number of discretization points, the aggregate results are robust and little additional accuracy is achieved.

[35]Importantly, as seen in Table D3 in online Appendix D, business cycle statistics for aggregate variables in the model with this TFP shock (and no disaster shock) process are broadly consistent with those in the data for the postwar period in the U.S.

$$\pi^{\eta} = \begin{pmatrix} \pi^{\eta}_{11} & \pi^{\eta}_{12} & \pi^{\eta}_{13} \\ \pi^{\eta}_{21} & \pi^{\eta}_{22} & \pi^{\eta}_{23} \\ \pi^{\eta}_{31} & \pi^{\eta}_{32} & \pi^{\eta}_{33} \end{pmatrix} = \begin{pmatrix} 0.908 & 0.090 & 0.002 \\ 0.045 & 0.910 & 0.045 \\ 0.002 & 0.090 & 0.908 \end{pmatrix}$$

I also add a stochastic process for economic disaster. There are three states involving different probabilities of economic disaster. These are current economic disaster ($\widetilde{d} = d$), high risk of economic disaster ($\widetilde{d} = h$) with a positive probability of disaster next period, and ordinary times ($\widetilde{d} = n$) with no probability of an economic disaster tomorrow. I assume that $\widetilde{d} \in \{d, h, n\}$ follows a Markov chain with

$$\pi^{\widetilde{d}} = \begin{pmatrix} \pi_{dd} & \pi_{dh} & \pi_{dn} \\ \pi_{hd} & \pi_{hh} & \pi_{hn} \\ \pi_{nd} & \pi_{nh} & \pi_{nn} \end{pmatrix} = \begin{pmatrix} \pi_{dd} & 1-\pi_{dd} & 0 \\ \frac{1-\pi_{dd}}{2} & \pi_{dd} & \frac{1-\pi_{dd}}{2} \\ 0 & 1-\rho_{\eta} & \rho_{\eta} \end{pmatrix} = \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0.094 & 0.906 \end{pmatrix}$$

where π_{ij} is the probability of transiting from disaster state i to disaster state j.

A time-varying aggregate risk of disaster is motivated by the rise in savings rates across households in the Great Recession, as seen in Table 14 below. A rise in savings rates by households may reflect a pronounced rise in precautionary savings in a recession.[36] While increased unemployment risk could potentially drive a rise in precautionary savings, Krueger et al. (2016) show that the increases in unemployment risk in recessions are not enough to deliver the rise in savings rates seen in the micro data. This motivates me to introduce a risk of an economic disaster, which implies large falls in wages and the return on high-yield illiquid assets, alongside an increase in unemployment risk. Though parsimonious, such a risk is consistent with findings in Kozlowski et al. (2017) that the left tail risk of aggregate shocks increased in the 2008 recession.[37]

Note that disasters are rare events, providing only limited empirical evidence to estimate the transition probability matrix directly from the data. To reduce the number of

[36] A rise in savings rates across households in the Great Recession is also documented in Krueger et al. (2016).

[37] Kozlowski et al. (2017) use capital returns data to estimate this.

parameters to calibrate, I impose parameter restrictions on $\pi^{\widetilde{d}}$.[38] First, I set the probability of staying in ordinary times equal to the persistence of TFP shocks, $\pi_{nn} = 0.906$. This is to make the persistence of an ordinary state similar to that of aggregate TFP shock. Second, I set the probability of going to a disaster state from an ordinary state and the probability of going to an ordinary state from a disaster state to zero ($\pi_{nd} = \pi_{dn} = 0$). Third, I impose the following symmetry conditions; $\pi_{hh} = \pi_{dd}$. Then, I only have one parameter to calibrate: the probability of staying in a high risk π_{hh}. I chose this parameter to match the 3 percent average probability of entering a disaster per annum estimated by Barro (2006).[39]

Crucially, given that disasters are severe recessions, there should be a correlation between the level of TFP and disaster risk. Thus, I assume that the economy does not face a risk of disaster unless the economy is in the lowest level of ordinary TFP, η_1. In other words, when the ordinary TFP level is either η_2 or η_3, the disaster state is always associated with ordinary times ($\widetilde{d} = n$) with no probability of disaster tomorrow. However, if ordinary TFP falls to its lowest level η_1, the economy faces a time-varying probability of disaster. Importantly, when ordinary TFP is η_1, economic disaster, should it occur, will result in an extraordinary drop in overall TFP to $\lambda\eta_1$, where $\lambda < 1$.[40] Such an additional drop in TFP in a disaster, $1 - \lambda = 0.2$, is calibrated to reproduce 30 percent of the observed decline in real GDP per capita in the U.S. during the Great Depression estimated by Barro (2006).[41]

Summarizing, Table 3 describes the five possible exogenous aggregate states of the economy and the effective TFP in each state. Note that the economy may have a low value of the current TFP, η_1, with either no ($\widetilde{d} = n$) or high probability ($\widetilde{d} = h$) of an economic

[38]This approach is also used in Glover et al. (2019) when they estimate their transition probability matrix for aggregate state involving a disaster.

[39]When the Markov chain for the disaster shock process is calibrated to match additional moments involving financial crises, the aggregate results are robust. These moments include the average duration of a banking crisis of 3.2 years and the average share of time spent in financial crisis around 7 percent of 1945 -2008 across advanced economies from Reinhart and Rogoff (2009). The current calibrated aggregate shock processes, including both TFP and disaster, imply two years of staying in a recession with high disaster risk as well as a 7.5 percent probability of entering such a recession.

[40]Disaster in my model is different from that in Gourio (2012) in the following two ways. First, in addition to a shock to TFP, Gourio (2012) also introduces a capital depreciation shock. However, I abstract from capital depreciation shock in my model economy. Second, in Gourio (2012), there is no correlation between ordinary TFP shocks and disaster risk.

[41]The 20 percent additional drop in TFP in a disaster is similar to the assumed 30 percent fall in TFP in a disaster in Gourio (2012). Ohanian (2001) also points out that there was an 18 percent decline in TFP between 1929 and 1933.

disaster next period.

Table 3: Aggregate state of the economy and overall TFP

aggregate state z	disaster $z_d \equiv (\eta_1, d)$	high risk $z_h \equiv (\eta_1, h)$	ordinary recession $z_n \equiv (\eta_1, n)$	normal (η_2, n)	boom (η_3, n)
$f(z)$	$\lambda\eta_1$	η_1	η_1	η_2	η_3

Combining the above two stochastic processes, the transition probability matrix for the possible five exogenous aggregate state becomes

$$\pi^z = \begin{array}{c} \begin{array}{ccccc} z_d & z_h & z_n & \eta_2 & \eta_3 \end{array} \\ \begin{pmatrix} \begin{pmatrix} \pi_{dd} & \pi_{dh} & \pi_{dn} \\ \pi_{hd} & \pi_{hh} & \pi_{hn} \\ \pi^{\eta}_{11}\pi_{nd} & \pi^{\eta}_{11}\pi_{nh} & \pi^{\eta}_{11}\pi_{nn} \end{pmatrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \\ \pi^{\eta}_{12} & \pi^{\eta}_{13} \end{matrix} \\ \begin{matrix} 0 & 0 & \pi^{\eta}_{21} \\ 0 & 0 & \pi^{\eta}_{31} \end{matrix} & \begin{matrix} \pi^{\eta}_{22} & \pi^{\eta}_{23} \\ \pi^{\eta}_{32} & \pi^{\eta}_{33} \end{matrix} \end{pmatrix} \end{array} = \begin{pmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0.25 & 0.5 & 0.25 & 0 & 0 \\ 0 & 0.085 & 0.823 & 0.090 & 0.002 \\ 0 & 0 & 0.045 & 0.910 & 0.045 \\ 0 & 0 & 0.002 & 0.090 & 0.908 \end{pmatrix}.$$

4.3 Unemployment and earnings shock process

Unemployment, for any household, is the result of a transitory shock that is i.i.d across households. It may last for part or all of a model period.

I select parameters governing this unemployment shock process to match the mean and median duration of unemployment, as well as the overall unemployment rate. The probability of unemployment and partial employment varies with the ordinary TFP shock, η_i. That is, π^i_e represents the probability of realizing working hours $e \in \{0, l_e, 1\}$ associated with TFP η_i. On top of this, the unemployment rate increases in a disaster. I use π^d_0 to match such a rise. The moments of calibration are summarized in Table 4.

First, the working period for a partially employed household l_e is chosen to match the median unemployment duration of 12 weeks between 1981 and 2016 in the CPS.[42] Next,

[42]Note that median unemployment duration is $\frac{12}{52} = 1 - l_e$.

Table 4: Unemployment shock process

parameters	value	moments
l_e	0.77	median unemployment duration (weeks) 12(1981-2016)
$\pi_l^1, \pi_l^2, \pi_l^3$	$0.140, 0.077, 0.049$	mean unemployment duration (weeks) 32(2008-2016), 24(1981-2016), 16(1981-2007)
$\pi_0^d, \pi_0^1, \pi_0^2, \pi_0^3$	$0.2022, 0.1345, 0.0325, 0.0056$	unemployment rates (%) 25(Great Depression), 10(2009), 5(2007), 3

Sources: CPS (unemployment duration) and BLS (unemployment rate). Sample periods are in parenthesis.

the probability of partial employment π_l^i is chosen to match the average unemployment durations over the three different sample periods in Table 4.[43] For example, π_l^2 is chosen to match the average unemployment duration between 1981 and 2016. π_l^1 and π_l^3 are chosen to match the average unemployment durations before and after the 2008 recession among these periods. Lastly, π_0^i is chosen to match the change in unemployment rates in the BLS. In particular, π_0^1 is chosen to explain a 5-percentage point rise in the unemployment rate from 2007 to 2009 (over the Great Recession) and π_0^d is chosen to match the unemployment rate of 25 percent during the Great Depression.[44]

I introduce a leptokurtic distribution of earnings shocks that matches the recent evidence from Guvenen et al. (2016) using the approach in Khan and Lidofsky (2019) and Kaplan et al. (2018). I assume that individual idiosyncratic earnings shock ε consists of two processes.

$$\log \varepsilon = \log \varepsilon_1 + \log \varepsilon_2$$

$$\log \varepsilon_i' = \rho_i \log \varepsilon_i + \zeta_i, \ i = 1, 2$$

where ζ_i is a jump process with an arrival rate of ν_i and $\zeta_i \sim N(0, \sigma_i^2)$. Khan and Lidofsky (2019) and Kaplan et al. (2018) estimate these earnings shock process to match eight key moments on male earnings data in Social Security Administration (SSA) reported in Guvenen et al. (2016) (see Table 6). The parameters, estimated for this earnings shock process in Khan and Lidofsky (2019), which is also an annual model, are summarized in

[43]The mean unemployment duration is calculated as $\frac{\pi_l(1-l_e)+\pi_u * 1}{\pi_l+\pi_u} * 52$ weeks.

[44]Measured TFP drops by 2 percent in the Great Recession. Thus, π_u^l is chosen to imply 10 percent of the unemployment rate when a TFP falls by 2 percent.

Table 5.

Table 5: Earnings shock process estimates

ρ_1	ρ_2	ν_1	ν_2	σ_1	σ_2
-0.05	0.97	0.016	0.695	2.61	0.22

Khan and Lidofsky (2019)

The estimated parameter values imply that the first shock is transitory with $\rho_1 = -0.05$, while the second shock process is highly persistent with $\rho_2 = 0.97$. It also implies that transitory shock is more volatile ($\sigma_1 = 2.61$) than persistent shock ($\sigma_2 = 0.22$) and transitory shock arrives less frequently, $\nu_1 = 0.016$, than the persistent shock, $\nu_2 = 0.695$.

Table 6 compares eight moments of the male earnings distribution in Guvenen et al. (2016) to those from the estimated earnings shock process. Khan and Lidofsky (2019) discretize this estimated continuous process as a Markov chain. I apply their method to discretize the income shock process into 15 values in my model. The implied 8 moments from this discretized process are listed in the right column of Table 6.

Table 6: Moments of the male earnings distribution

	Guvenen et al. (2016)	continuous	discrete
variance annual log earnings	0.70	0.70	0.70
variance 1 year change	0.23	0.26	0.26
variance 5 year change	0.46	0.38	0.63
kurtosis 1 year change	17.80	17.81	17.7
kurtosis 5 year change	11.60	11.60	11.65
fraction 1 year $\leq$ 10 percent	0.54	0.53	0.56
fraction 1 year $\leq$ 20 percent	0.71	0.72	0.66
fraction 1 year $\leq$ 50 percent	0.86	0.96	0.94

Khan and Lidofsky (2019)

4.4 Remaining parameters

I set the coefficient of relative risk aversion (γ) to 2 and the IES($\frac{1}{\sigma}$) to 1.5.[45] Following Hosseini (2015), θ_s is chosen to match the replacement rate of 45 percent of average pre-tax

[45] Gourio (2012) shows that it is important to have the IES greater than 1 in a model with a disaster risk to reproduce a countercyclical risk premium as seen in the data.

earnings in the steady-state. I choose $\theta_u = 0.25$ to match 43.5 percent for a replacement rate of unemployment benefits paid for a maximum of 26 weeks (Nakajima (2012)). Labor income is taxed at 27 percent (Domeij and Heathcote (2004)). I choose τ_a to imply a 25 percent capital income tax rate in the steady-state.[46] The capital share of output is $\alpha = 0.36$ and the annual depreciation rate of capital is $\delta = 0.069$. Please see Table 7 and 8 for the summary of parameters.

[46]Elenev et al. (2018) estimate a capital income tax rate of 20 percent from government corporate tax revenue as a share of GDP using BEA data. In Jermann and Quadrini (2012), the average tax rate is 35 percent. I choose a value in this range.

Table 7: Parameters set externally

parameters	value	description
γ	2.0	coefficient of relative risk aversion
σ	$\frac{1}{1.5}$	inverse of IES
α	0.36	capital share of output(NIPA)
δ	0.069	depreciation rate(NIPA)
θ_u	0.25	replacement rate of unemployment benefits (Nakajima, 2012)
θ_s	0.4	replacement rate of social security benefits (Hosseini, 2015)
τ_n, τ_a	0.27, 0.25	labor and capital income taxes
ρ_η, σ_η	0.906, 0.015	TFP shock process

Table 8: Parameters calibrated

parameters	value	moments to match	data	model
ξ_0	0.006	capital to output ratio	2.66	2.66
ξ_1	0.04	share of liquid asset to output	0.35	0.35
β_L	0.886	wealth Gini	0.78	0.78
β_M	0.907	fraction of households holding illiquid wealth	0.73	0.73
β_H	0.971	fraction of households with positive illiquid wealth but no liquid wealth	0.32	0.20
$\underline{b}$	-0.03	fraction of households holding zero or negative net worth	0.103	0.16
l_e	0.77	median unemployment duration (1981-2016)		
$\pi_l^1, \pi_l^2, \pi_l^3$	$0.140, 0.077, 0.049$	mean unemployment duration	see Table 4	see Table 4
$\pi_0^d, \pi_0^1, \pi_0^2, \pi_0^3$	$0.2022, 0.1345, 0.0325, 0.0056$	unemployment rates		
λ	0.8	drop in GDP per capita during the Great Depression (Barro, 2006)	0.30	0.30

5 Numerical method

In this section, I briefly describe the numerical method implemented to solve the model economy. For more technical details and accuracy of the method, please see Appendix E.

I first develop a two-stage approach that uses an intermediate value function to solve households' savings decisions with two assets. Here, I define an intermediate value function over cash-on-hand, the future stock of illiquid assets, and idiosyncratic types of households. In the first stage, households make their portfolio adjustment decisions and savings in illiquid wealth to maximize this intermediate value function. Note that if a household pays a fixed adjustment cost, its cash-on-hand includes the current holdings of illiquid assets. However, if a household chooses not to adjust, it cannot cash in the current holdings of illiquid assets, and only liquid assets and labor (social security) income are available. In the second stage, households choose consumption and liquid savings, given their illiquid asset choice and the remaining cash value from the first stage.[47]

Second, to solve the model with aggregate uncertainty, I extend the backward induction method of Reiter (2002, 2010).[48] This involves generalizing the method to solve an OLG economy with a bivariate cross-sectional distribution of continuous endogenous state variables. The backward induction method directly solves cross-sectional distributions, called *proxy distributions*, across aggregate and approximate aggregate states without imposing any parametric restrictions on distributions. This proxy distribution is solved using *distribution selection function* (DSF), which maps aggregate states to cross-sectional distributions subject to moment consistency conditions. With proxy distributions solved, backward induction simultaneously solves for households' decision rules and an end-of-period

[47]Technically, the endogenous grid method (EGM) can be implemented in the second stage, which reduces computation time tremendously. However, in my model economy, the finite difference approximation for the derivative of the value function is not very accurate with a discrete choice. Thus, I used the golden section search to find the optimizers. However, by introducing random adjustment costs instead of deterministic adjustment costs, we can implement EGM instead.

[48]Both Krusell and Smith (1998) and Reiter (2002, 2010) solve households' decisions over approximate aggregate states which summarize infinite-dimensional cross-sectional distributions using a finite vector of moments of the distribution. However, Reiter (2002, 2010) does not impose any parametric forecasting rule for the law of motion for the approximate aggregate state. Finding this forecasting rule in Krusell and Smith (1998) requires repeated long-period simulations. Moreover, this simulation step is exceptionally costly in my model economy as equilibrium prices are not pre-determined by the beginning-of-period aggregate state of the economy as in Krusell and Smith (1998).

distribution, implying a future approximate aggregate state consistent with households' expectations. This enforces consistency between individual behavior and the aggregate law of motion in each step.[49] Lastly, I simulate the model economy and weight simulated distributions using an inverse quadratic to update the (reference) distributions for the DSF.

The backward induction method has particular advantages over other existing methods in terms of two things. First, this method does not linearize the model and thus does not rely on *certainty equivalence* in terms of aggregate shocks, as in Ahn et al. (2018). This allows the model to introduce a direct role of aggregate risk to the household decisions and its distribution. Second, by allowing the distribution of households to vary in potentially rich ways, this method is robust to solve the model economy with multiple assets, especially when households have significant changes in their asset holdings following a large aggregate shock.

6 Quantitative results

In this section, to explore the aggregate effects of heightened disaster risk, I compare the dynamics of aggregate quantities across two different recessions. In the first, only TFP falls while, in the other, disaster risk rises with a fall in TFP. I present aggregate results in the two model economies. The first model is the benchmark economy with both liquid and illiquid assets. The second model is an otherwise comparable Aiyagari (1994) economy only with a *single liquid physical capital asset*. Comparing these two economies, I show that household differences in the holdings of both liquid and illiquid assets is an important channel that amplifies the effects of a rise in disaster risk, thereby explaining the sharp fall in aggregate consumption and investment seen during the Great Recession.

[49]By contrast, Krusell and Smith (1998) solve households' decision rules, assuming an aggregate law of motion, and then update this forecasting rule by simulating the model. This repeats until the consistency between individual decision rule and the aggregate law of motion is achieved.

6.1 Single asset economy

I first discuss how a rise in disaster risk in the second model, an economy with a single asset, changes the aggregate responses compared to a shock to only current TFP. In this economy, all households save using the same asset, *liquid physical capital asset*, and there is no portfolio adjustment cost. However, to ensure comparability to the benchmark economy, with both liquid and illiquid assets, I continue to assume that the investment firm faces convex capital adjustment costs.

The solid black lines in Figure 4 show the impulse responses to only a negative TFP shock, with no change in disaster risk.[50] Here, TFP falls by 2 percent for the first two periods and then monotonically returns to its mean value, following its AR(1) process. The assumed 2 percent change in TFP is the observed fall between 2007Q4 and 2009Q2 levels.[51] Note that this also leads to a rise in the unemployment rate to 10 percent, consistent with the data.

As seen in Figure 4, the responses in aggregate quantities to a persistent negative TFP shock are familiar from those seen in a standard real business cycle model. For example, there are declines in consumption and investment, and the fall in aggregate consumption is U-shaped. Overall consumption and investment fall 2.5 and 8.8 percent, respectively, compared to their levels in the absence of shocks.[52]

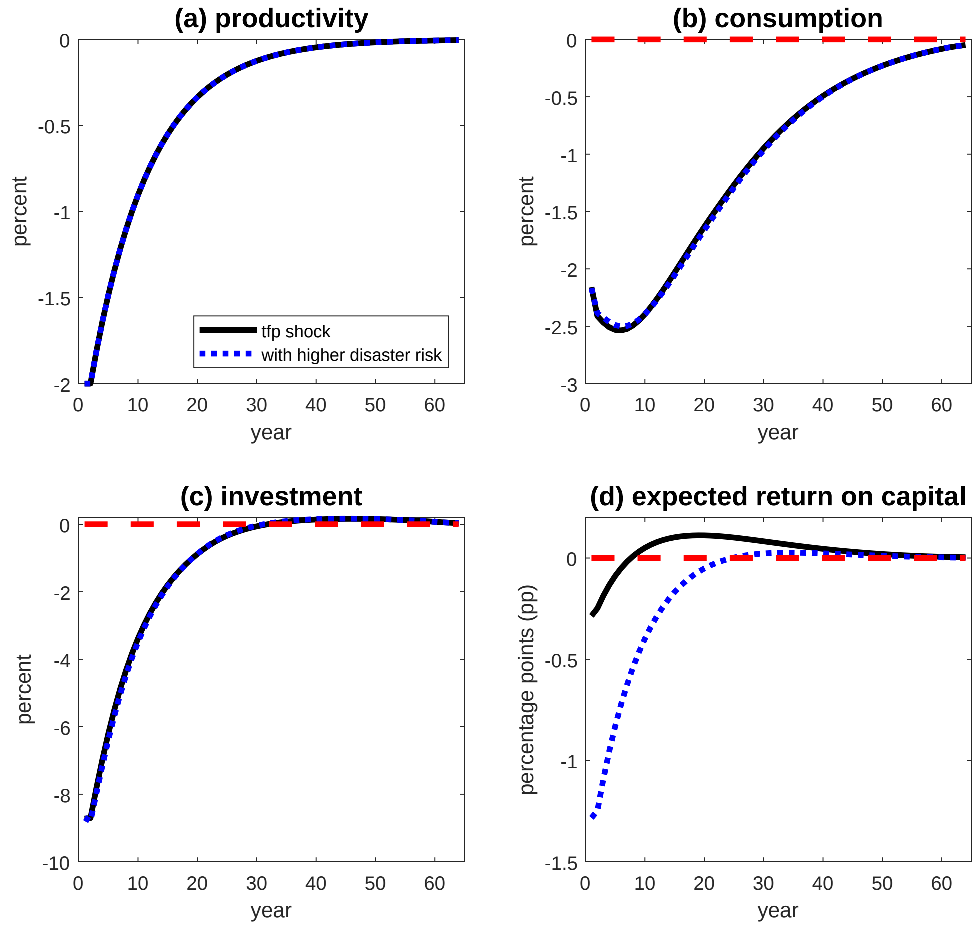
Next, the dashed blue lines in Figure 4 show the impulse responses to a combined TFP shock with a rise in disaster risk. Note that as TFP returns to its simulated mean over time,

[50] Please see Figure F1 in Appendix F for impulse responses of all aggregate variables.

[51] Fernald and Matoba (2009) also find that the annual TFP growth fell by 2 percent in the Great Recession. Moreover, after adjusting to the lower utilization of labor and capital, they find that utilization-adjusted TFP rose for the Great Recession.

[52] Krueger et al. (2016) also study the implications of cross-sectional wealth inequality for aggregate dynamics. They find that cross-sectional inequality in wealth decreases aggregate consumption by 2.4 percentage points compared to a 1.8 percentage point drop in a representative-agent economy and a 1.9 percentage point drop in the Krusell-Smith economy which has less wealth inequality. Note that, in my single asset economy, consumption falls by 2.5 percentage points, even with a rise in disaster risk, which is similar to the prediction by Krueger et al. (2016).

Figure 4: Impulse responses in a single asset economy



Notes: Expected return on capital$= \frac{E[p'+d']}{p} - 1$. Figure 4 shows the aggregate dynamics of a single asset economy with disaster risk in response to a 2 percent TFP drop (solid black line) and to a 2 percent TFP drop alongside a rise in disaster risk (dashed blue line).

the heightened probability of economic disaster simultaneously subsides.[53] Importantly, the aggregate dynamics of consumption and investment, following a rise in disaster risk, are

[53]Technically, the 2 percent drop in TFP lies between the two discretized points, η_1 and η_2. As explained in the calibration section, with the TFP level η_2, the economy does not face a disaster risk. Thus, over the simulation, I must interpolate a high-risk recession with normal time without disaster risk (see Table 3). This implies that the economy recovers from a high-risk recession to mean TFP, η_2, without going through an ordinary recession.

close to those from a shock to TFP alone.[54]

A rise in disaster risk leads to a large negative wealth effect. If an economic disaster occurred, the economy would experience a catastrophic fall in TFP, reducing wages and returns to capital and increasing unemployment risk. A higher probability of such a disaster, in a recession, reduces households' expected future income.[55] All else being equal, this negative wealth effect leads households to increase their precautionary savings and decrease consumption.

The minuscule aggregate effect of an increased probability of disaster in this Aiyagari economy is the result of a large substitution effect that offsets the negative wealth effect. The rise in precautionary savings, discussed above, must translate into investment in physical capital in this economy. This pushes down the equilibrium return to capital. For example, in this single asset economy, a rise in disaster risk lowers the expected return to capital by more than 1.3 percentage points. Without illiquidity in assets, households strongly respond to a fall in the expected return to capital by decreasing their savings and increasing consumption. As a result, the substitution effect substantially offsets the negative wealth effect arising from a rise in disaster risk.[56] Overall, there is little change in aggregate dynamics compared to a TFP shock driven recession.

6.2 Two assets economy

I now explore how aggregate dynamics respond to a rise in disaster risk when households differ in their holdings of both liquid and illiquid assets. Figure 5 presents impulse responses, comparable to those in Section 6.1. As above, these follow from a 2 percent drop in only TFP (solid black lines), and to both a TFP shock and a rise in disaster risk

[54]This is not driven by a less dispersed distribution of wealth in the single asset economy. As shown in Table 2, the single asset economy reproduces a very similar distribution of wealth to that seen in the benchmark economy.

[55]This is consistent with the empirical findings in De Nardi et al. (2012). Using the Michigan Survey of Consumers, they find a significant fall in expected income growth across households during the Great Recession. Specifically, the Michigan Survey of Consumers (MSC) asks the following questions to households: 1) "During the next 12 months, do you expect your income to be higher or lower than during the past year?" 2) " By about what percent do you expect your income to (increase/decrease) during the next 12 months?".

[56]Such a large interest rate elasticity of aggregate nondurable consumption in heterogeneous agent economy is also discussed by Guerrieri and Lorenzoni (2017).

(dashed blue lines).[57] Here, I assume that the government provides an elastic supply of liquid assets. Thus, in this economy, the return on safe liquid assets is held constant at one percent.[58,59]

As seen in Figure 5, following a negative TFP shock with no concurrent rise in disaster risk, the largest declines in consumption and investment are 2.5 percent and 11.8 percent, respectively. While the largest drop in aggregate consumption is similar to that in the single asset economy of Section 6.1, investment falls 3 percentage points more in the economy with two assets. Given that illiquid assets are capital in the model, such an additional drop in investment is driven by the fact that households increase their precautionary savings using safe liquid assets.[60] Physical capital is both risky and, given their costs of adjustment, illiquid. This makes them less useful for smoothing consumption changes.

Importantly, in contrast to the single asset economy, Figure 5 shows that, when we introduce a higher likelihood of economic disaster, there are further decreases in aggregate consumption and investment compared to what we saw from those a TFP shock, alone. First, with a rise in disaster risk, consumption falls by 3.5 percent from its simulated mean while the largest decline, following a TFP shock only, is 2.5 percent.[61] As argued by Krueger et al. (2016), such an additional percentage point drop in aggregate consumption is both large and significant.

A greater fall in aggregate consumption is driven by the large negative wealth effect and smaller substitution effect, when compared to the single asset economy. As mentioned before, a rise in disaster risk reduces households' expected future income. For young and

[57]Please see Figure F2 in Appendix F for impulse responses of all aggregate variables.

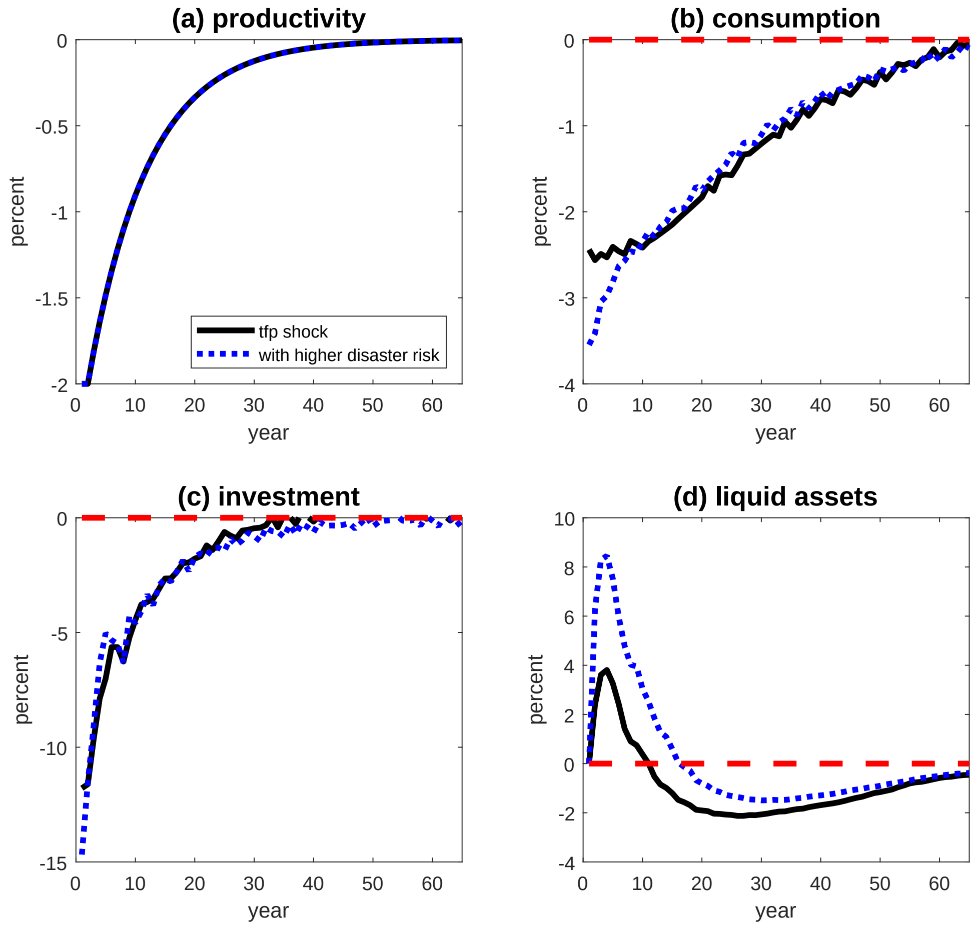
[58]Note that, while the return on liquid savings is held constant in equilibrium, this is consistent with general equilibrium characterized by a government policy that adjusts the supply of liquid assets to maintain the constant return.

[59]The alternative two assets economy with a fixed supply of liquid assets is solved in the online Appendix A. Though the aggregate impact of a rise in disaster risk is weakened with a fixed supply of liquid assets, a rise in disaster risk still explains a 0.5 percentage point larger drop in aggregate consumption compared to that following only a TFP shock. Note that each of these assumptions about government policy over the business cycle serves as a benchmark. Over the Great Recession, real interest rates on safe, liquid assets fell markedly while, at the same time, quantitative easing led to a considerable rise in their supply.

[60]Note that precautionary savings rise with a fall in TFP as the unemployment risk rises.

[61]Note that, given the correlation of the level of ordinary TFP and disaster risk, as the TFP recovers to its simulated mean, the probability of a disaster next period disappears. That is why we see similar responses of the aggregate variables after a certain simulation period across the two recessions.

Figure 5: Impulse responses in a two-assets model (r_f fixed)



Notes: Figure 5 shows the aggregate dynamics of the benchmark economy with both liquid and illiquid assets following a 2 percent TFP drop (solid black line) and a 2 percent TFP drop alongside a rise in disaster risk (dashed blue line). The vertical axis measures the percent changes of each variable from its simulation mean.

wealth-poor households, whose primary income source is labor earnings, expected future income falls because of lower expected wages and higher unemployment risk. For old and wealthy households, whose primary income source is capital, their expected future income is lower because of a fall in the expected return to illiquid assets. These negative wealth effects increase households' precautionary savings, pushing down their consumption over and beyond the changes that follow a negative shock to TFP.

However, in contrast to the single asset economy, the costly adjustment of illiquid assets in the benchmark economy makes households holding high-yield illiquid wealth less responsive to the fall in the expected return on savings. Indeed, only 16-18 percent of households are actively adjusting their holdings of illiquid assets in a model period (see Table 15). This weakens the substitution effect compared to a single asset economy. As this effect tends to partly offset the wealth effect, a smaller average substitution effect amplifies the changes seen from a rise in disaster risk.

Figure 5 also shows that a higher disaster risk leads to a larger drop in aggregate investment in physical capital and a sharp increase in savings in liquid assets, compared to a TFP-shock driven recession.[62] This is because of a rise in precautionary savings in safe liquid assets and the related portfolio adjustment behavior of some households, away from illiquid savings into safe liquid assets. Note that this implies that, when the government increases liquidity in the economy to stimulate consumption in a recession, households use these extra liquid assets to increase their precautionary savings instead of spending. This suggests that it is essential to understand the fundamental shock that causes a recession, especially when studying the effects of fiscal stimulus.

Table 9: Peak-to-Trough declines: 2008 U.S. Recession and model

	GDP	I	N	C
data	5.59	18.29	5.99	3.64
single asset economy	5.39	8.79	5.07	2.50
two asset economy	5.52	14.7	5.07	3.55

Notes: Table 9 shows peak-to-trough declines between 2007q4 and 2009q2 in the first row. The second row is a single asset economy. The third row is the benchmark economy with two assets, where the return on liquid wealth is fixed.

Summarizing, in Table 9, I compare the peak-to-trough declines in aggregate quantities from the data to those seen in the two model economies in a recession involving both a fall in TFP and a rise in disaster risk. Table 9 shows that, in a recession with a heightened probability of disaster, a single asset economy experiences a 2.5 percent drop in aggregate

[62] The observed sharp increase in safe liquid assets between 2007 and 2010 is documented in Bayer et al. (2019) using the 2007 and 2010 SCF data.

consumption and a 8.8 percent drop in investment relative to their simulated means. [63] By contrast, the data shows that the U.S. economy experienced much sharper falls in the 2008 recession; aggregate consumption fell by 3.6 percent and investment fell by 18 percent. The last row of Table 9, introducing an additional illiquid asset that has risky returns, explains declines in aggregate consumption and investment much closer to those in the data. In this two-assets economy, consumption falls by a 3.5 percent, and investment falls by around 15 percent. Comparing these results to the one asset model, we see that a rise in disaster risk, in a recession, does not, by itself, explain the anomalous declines in aggregates seen in the data. An important channel to amplify such a shock is household heterogeneity in the holdings of both liquid and illiquid assets.[64]

6.3 Sensitivity Analysis

The previous section shows that households' heterogeneity in both liquid and illiquid assets amplifies a rise in disaster risk, driving further declines in aggregate quantities compared to the effects of a negative TFP shock by itself. In this section, I explore the robustness of these results across alternative economies that vary in their preference parameter values.

First, in Table 10, I show how different preference parameters alter the pre-recession

[63] In model economies, GDP falls the most on the initial impact date. Thus, peak-to-trough fall in GDP is determined by the exogenous fall in aggregate labor supply and the beginning period-of-capital stock - steady-state level of capital- when the aggregate shock hits. This is why the largest fall in GDP is similar across the economies.

[64] In Gourio (2012)'s representative agent economy with safe and risky assets, a higher disaster risk leads to a sharp fall in investment and an increase in consumption. The fall in investment is driven by the capital destruction shock in a disaster, which I abstract from my model economy. A depreciation shock makes capital investment risky, forcing households to adjust their portfolio towards safe assets in response to a rise in disaster risk. However, as discussed in Gourio (2012), without capital depreciation shock in a disaster, a rise in disaster risk, with no change in TFP, counter-factually leads to an investment and output boom as household increases its precautionary savings in the capital. I also solve an infinite-lived representative agent economy with liquid and illiquid assets and disaster risk in online Appendix B and obtain similar results. I find that a rise in disaster risk actually dampens the falls in investment and liquid assets while the largest fall in consumption is similar compared to a TFP-shock only recession. This is a result of capital becoming a relatively safe liquid asset in this representative-agent economy. See online Appendix B for more details.

Table 10: Distributions of net worth, illiquid and liquid assets in different models

Different version of models	Net worth						Gini		
	Q1	Q2	Q3	Q4	Q5	≤ 0	wealth	illiquid	liquid
(1) Benchmark	-0.0	0.6	3.1	14.8	81.7	17.0	0.78	0.77	0.85
(2) No β heterogeneity	6.8	24.9	40.6	91.2	100	3.0	0.59	0.59	0.86
(3) CRRA preference	7.0	24.5	40.2	91.1	100	1.0	0.60	0.60	0.76
(4) High risk aversion	5.9	21.2	35.6	88.8	100	3.2	0.55	0.55	0.77

Notes: Table 10 shows quintile distributions of net worth. It also reports the share of households with zero or negative net worth and the Gini coefficients for net worth, illiquid assets, and liquid assets in different versions of the model: (1) Benchmark economy ($\gamma = 2.0$, $IES = 1.5$), (4) No heterogeneity in discount factors ($\gamma = 2.0$, $IES = 1.5$), (3) CRRA preference ($\gamma = 2.0$, $IES = 0.5$), and (4)High risk-aversion parameter ($\gamma = 4.0$, $IES = 1.5$).

cross-sectional distribution of wealth across alternative models.[65] For example, wealth inequality is significantly reduced when I eliminate households' differences in their subjective discount factors. This exercise, shown in the second row, maintains the IES and risk-aversion parameters at their benchmark values. When I also move to CRRA preferences by reducing the IES to 0.5, holding relative risk aversion to its benchmark value ($\gamma = 2$), wealth inequality largely falls. Note that, with the IES less than unity, households become less willing to substitute consumption intertemporally in response to changes in the returns to savings. Given that liquid assets are more desirable for consumption smoothing than illiquid assets, this leads households to hold a higher fraction of their wealth in liquid assets, compared to the benchmark economy. This reduces liquid wealth inequality. In the fourth row, a higher relative risk-aversion parameter than the benchmark value also decreases wealth inequality. Now, highly risk-averse households sharply increase their precautionary savings in safe liquid assets.

In Table 11, I report the associated peak-to-trough declines in investment and consumption following a negative TFP shock, and those following a combined TFP shock and rise in disaster risk, across different models.[66] In the last two columns, I show the additional

[65] In this sensitivity analysis, I explore how different preference parameter values alter the results, including the steady-state distribution. Thus, I do not re-calibrate the steady-state to isolate the effect of a change in a parameter of interest. This avoids offsetting effects caused by changes in other parameters.

[66] I do not report aggregate results for GDP as the largest decline in GDP happens on the initial impact date. The decline in the initial impact date is purely determined by the exogenous fall in aggregate labor supply and the beginning period-of-capital stock before aggregate shocks hit.

Table 11: Peak-to-Trough declines in different models

Different version of models	TFP only		TFP+ disaster risk		difference	
	I	C	I	C	ΔI	ΔC
(1) Benchmark ($\gamma = 2.0$, $IES = 1.5$)	11.8	2.56	14.7	3.55	+2.9	+1.0
(2) No β heterogeneity ($\gamma = 2.0$, $IES = 1.5$)	11.1	2.86	12.4	3.46	+1.3	+0.6
(3) CRRA preference ($\gamma = 2.0$, $IES = 0.5$)	13.4	2.82	16.3	3.90	+2.9	+1.1
(4) High risk aversion ($\gamma = 4.0$, $IES = 1.5$)	9.46	2.61	12.7	3.25	+3.2	+0.6

Notes: Table 11 shows peak-to-trough declines in investment (I) and consumption (C) in different models across two types of recessions, one with only a TFP shock and the other with both a TFP shock and heightened disaster risk. The last two columns show the additional percentage points falls, peak-to-trough, driven by a rise in disaster risk, compared to a TFP shock-driven recession.

percentage point falls in investment and consumption when I add a rise in disaster risk to a TFP shock. Importantly, the aggregate results in Section 6.2 are robust across different models. As seen there, in contrast to the single asset economy, a rise in disaster risk further decreases aggregate investment and consumption compared to a TFP shock by itself, in all alternative economies.

In the first alternative economy without heterogeneous time discount factors, a rise in disaster risk has less impact on aggregate quantities than in the benchmark economy. The second row of Table 11 shows that the additional fall in investment, from a rise in disaster risk, falls to 1.3 percentage points from 2.9 percentage points in the benchmark economy. Also, the additional fall in consumption drops from 1.0 percentage point to 0.6 percentage points. This is because the alternative economy has fewer wealth-rich and wealth-poor households than the benchmark economy. When wealthy households disinvest in illiquid assets following a rise in disaster risk, aggregate investment falls. Fewer households of this type, therefore, dampens the effect of disaster risk on investment. Moreover, wealth-poor households are those who sharply increase their precautionary savings in a recession. Thus, with less wealth-poor households, aggregate consumption response to a rise in disaster risk is mitigated.

Turning to CRRA preferences in row 3 of Table 11, despite lower wealth inequality, a high probability of economic disaster has similar effects on aggregate investment and consumption to those in the benchmark model with recursive preferences. This is due to the fact that households are less willing to tolerate consumption fluctuations with a lower IES. Thus, when households see a higher probability of an economic disaster that reduces the certainty equivalent of future expected income, they react more strongly by increasing safe liquid assets. This leads to a more significant fall in both consumption and investment in physical capital. As a result, though wealth is less dispersed with CRRA preferences than in the benchmark economy, a rise in disaster risk has similar impact on consumption and investment.[67]

Lastly, in row 4, the relative risk aversion parameter increases to $\gamma = 4$. Now, the additional fall in investment arising from higher disaster risk rises to 3.2 percentage points from 2.9 percentage points in the benchmark economy. This is largely driven by highly risk-averse households responding more strongly to a rise in disaster risk by disinvesting illiquid assets to smooth consumption in a recession.

6.4 Household dynamics

In this section, I explore household behavior over the Great Recession, compared to the pre-recession. Specifically, I examine changes in disposable income, consumption, and expenditure rates, over wealth quintiles. As argued by Krueger et al. (2016), these moments serve as important measures to evaluate the quantitative fit of heterogeneous household business cycle models.

The analysis in this section shows that the benchmark model is broadly consistent with the slowdown in the growth rates of disposable income and consumption. Most importantly, the model is consistent with the rise in household savings rates seen during the Great Recession. Next, it suggests a relatively important role for the top of the distribution of households in determining changes in aggregate investment.

[67]In a fully re-calibrated model with CRRA preference that reproduces a similar distribution of wealth as in the benchmark economy, a rise in disaster risk has larger effects on aggregate consumption and investment than in the benchmark economy (see online Appendix C).

6.4.1 Distribution of income, consumption and expenditure rates

I document the observed growth rates of disposable income and consumption as well as changes in expenditure rates, over wealth quintiles, using the PSID data.[68] Here, I define net worth as total assets minus total debt. Disposable income is measured as the sum of labor income, unemployment benefits and income from assets minus federal and state income taxes.[69] Total expenditure is the total spending on nondurable goods and services.[70]

In this analysis, I focus on working households with a head who is between 25 and 65 years old. My focus on working households allows the analysis to be comparable to that in Krueger et al. (2016) – the only other existing business cycle model that conducted the same exercise for working households.[71]

Table 12 summarizes the annualized growth rates of the average level of disposable income and consumption between 2005 and 2007 over 2005 wealth quintiles.[72] It also presents percentage point differences in expenditure rates. For the corresponding moments in the model economy, I simulate 2 million households for three years over a normal time characterized by the mean value of TFP and no disaster risk. Note that none of the empirical moments are targeted in the model. In order to compare household behavior with the only other model that conducted the same analysis, I also present the corresponding model simulated moments in Krueger et al. (2016).

Table 12 shows that the model economy is consistent with several stylized facts seen in

[68] See Appendix A for details on data and sample selection.

[69] Labor earnings include after-tax wages and salaries, bonuses, overtime, tips, commissions, and transfers. Taxes are calculated using the NBER TAXSIM calculator.

[70] Items on expenditure surveyed in the PSID are listed in Appendix A. I also compare the PSID constructed expenditure data to BEA data in Appendix A and find that spending on each item as a share of total expenditure in the PSID is broadly comparable to that in the BEA.

[71] Appendix G presents results from the same exercise including households with a head who is between 25 and 85 years old. As previously shown in Figure 1, the benchmark economy predicts a counterfactually rapid dissaving after retirement by households. Thus, including retirees in this exercise overestimates the average age in the bottom quintile of the wealth distribution in the model, relative to the data, making pre-recession moments less consistent with those in the data (see Table G2).

[72] Following Krueger et al. (2016), I keep households in each quintile fixed and calculate the annualized percentage change of the averages between two years. The PSID data moments in this section are broadly consistent with those documented in Krueger et al. (2016). Some discrepancies in moments come from differences in sample selection. Krueger et al. (2016) look at households with heads between 25 and 60 years of age while I look at those with heads between 25 and 85 years old.

Table 12: Annualized growth rates of variables across wealth quintiles before the Great Recession 2005-2007

2005–2007	Disp. Income (%)			Expenditure (%)			Exp. Rate ($\Delta p.p.$)		
NW Quintiles	PSID	Model	KMP	PSID	Model	KMP	PSID	Model	KMP
Q1($poor$)	6.3	6.4	7.2	6.4	11.2	6.7	0.1	3.3	-0.4
Q2	4.8	4.7	3.1	3.8	4.7	3.6	-0.6	0.0	0.5
Q3	2.6	2.4	1.6	4.4	1.9	2.5	0.9	-0.4	0.8
Q4	3.9	-1.3	0.5	4.2	-0.7	1.7	0.1	0.5	1.2
Q5($wealthy$)	-2.9	-2.9	-1.0	3.6	0.2	0.5	3.3	2.9	1.4

Notes: This shows annualized growth rates of average disposable income and expenditure for households with a head less than 65 years old. It also reports percentage point changes in expenditure rates from the 2005-2007 PSID and the benchmark economy during normal times. It also shows the corresponding moments in Krueger et al. (2016) (denoted as KMP).

the PSID before the Great Recession. First, the growth rate of disposable income declines with a household's wealth quintile. For example, in the data, disposable income grows by 6.3 percent for the lowest wealth group while it falls by 3 percent for the wealthiest. The model economy successfully reproduces these growth rates of disposable income. Second, the expenditure data show that the growth rate of consumption decreases by wealth group except at the second quintile.[73] Similarly, in the model, the growth rates of consumption do generally fall in wealth. Lastly, the model reproduces the general increase in expenditure rates seen in the data. Some channels relevant for explaining consumption-savings behavior of households, such as bequests or health shocks, are missing in the model. It also abstracts from trend growth, which may cause a discrepancy between model and data. However, as none of these moments were targeted, the model explains household spending, over wealth, arguably well.

Table 13 shows changes in household income and spending, comparable to Table 12, but for the 2007 to 2011 period. Wealth quintiles are fixed to 2007. The model moments are generated by a drop in TFP alongside a rise in disaster risk.[74] Given that my focus is

[73] Krueger et al. (2016) also find the empirical consumption growth rates falling in wealth over the same sample period.

[74] I used the 2007 and 2011 data for the Great Recession as in Krueger et al. (2016). Note that the PSID is a biennial data including surveys for 2007, 2009 and 2011. Given that some of the variables, such as income, are retrospective, using the 2009 data, instead of the 2011 data, can miss the impacts of the 2008 recession on those variables.

to explore how households were affected in a recession, compared to normal times, Table 14 also presents differences in the annualized changes of variables between pre-recession (2005-2007) and recession (2007-2011) periods.

Table 13: Annualized growth rates of variables across wealth quintiles during the Great Recession 2007-2011

2007–2011	Disp. Income (%)			Expenditure (%)			Exp. Rate ($\Delta p.p.$)		
NW Quintiles	PSID	Model	KMP	PSID	Model	KMP	PSID	Model	KMP
Q1(*poor*)	5.5	3.9	4.9	1.6	7.1	4.5	-2.4	2.2	-0.4
Q2	3.7	2.4	0.3	2.7	1.3	1.2	-0.7	-1.0	0.8
Q3	2.0	0.1	-2.4	1.2	-0.8	0.0	-0.6	-0.8	2.2
Q4	1.3	-3.3	-4.0	0.3	-2.8	-1.5	-0.7	0.4	3.2
Q5(*wealthy*)	-0.3	-4.9	-6.4	0.1	-1.6	-3.5	-0.1	3.4	4.6

Notes: This shows annualized growth rates of average disposable income and expenditure. It also reports percentage point changes in expenditure rates from the 2007-2011 PSID and the benchmark economy during the Great Recession.

Table 14: Changes in growth rates of variables before and during the Great Recession

Difference	Disp. Income (%)			Expenditure (%)			Exp. Rate ($\Delta p.p.$)		
NW Quintiles	PSID	Model	KMP	PSID	Model	KMP	PSID	Model	KMP
Q1(*poor*)	-0.8	-2.5	-2.3	-5.1	-4.2	-2.2	-2.5	-1.1	0.0
Q2	-1.1	-2.3	-2.8	-0.8	-3.5	-2.4	0.2	-1.0	0.3
Q3	-0.7	-2.3	-4.0	-3.4	-2.7	-2.5	-1.4	-0.4	1.4
Q4	-2.6	-2.0	-4.5	-4.7	-2.1	-3.2	-1.1	-0.1	2.0
Q5(*wealthy*)	-2.6	-2.0	-5.4	-4.3	-1.8	-4.0	-3.5	0.4	3.7

Notes: This shows changes in annualized growth rates of average disposable income and expenditure, and percentage point changes in expenditure rates from the PSID and the benchmark economy between normal times (2005-2007) and recession (2007-2011).

Table 14 first shows that the model economy, consistent with data, predicts a slowdown in the growth rates of disposable income and consumption across all wealth quintiles in the recession.[75] Second, during the Great Recession, consumption rates fell for most households

[75] In the PSID, growth rates of income decline the most for the top of the distribution of households compared to the bottom of the distribution of households. This might be the result of a sharp fall in asset prices during the Great Recession, which is missing in my model.

compared to their pre-recession levels. In other words, *saving rates increased*.[76] While the model economy does not explain enough of the observed increase in savings rates seen in the data, it nevertheless captures the trend increase seen in most wealth groups. While I only analyze working household's behavior in this section, Appendix G, shows that these results for changes in growth rates and expenditure rates, are robust when I include retirees.

The rise in savings rates in a recession is hard to explain in a model with only a negative shock to current TFP. As shown in the last column of Table 14, Krueger et al. (2016) show that, in the recession with a shock to TFP and a rise in unemployment risk, consumption rates sharply increase and saving rates fall for all quintile groups. This is because, their recession implies a monotone drop in income. Consumption smoothing motives then lead households to decrease their consumption less than the realized fall in their income. This leads to a counterfactual rise in consumption rates and a fall in savings rates. Note, however, that savings rates actually increased for most households during the Great Recession. This suggests a strong precautionary savings motive. In my model economy, a higher probability of a further economic downturn increases households' precautionary savings. This allows my model to better explain the fall in consumption rates, thus the rise in savings rates, seen in the Great Recession.

Lastly, it is worth noting that households with relatively small levels of wealth play a significant role in explaining the dynamics of aggregate consumption despite holding a negligible share of wealth. These households, with high MPCs, strongly respond to aggregates shocks. They decrease their consumption and increase savings in the safe liquid asset. As shown in Table 14, households in the first quintile exhibit a relatively sharp drop in the growth rate of consumption, and in expenditure rates themselves in the recession.[77] As also pointed out by Krueger et al. (2016), this implies a crucial role for the bottom of

[76] Krueger et al. (2016) also document the rise in savings rates by households in the 2008 recession and find 4 percentage points fall in consumption rates by the bottom quintile. Contrary to the fall in expenditure rates by all quintiles in Krueger et al. (2016), my data shows an anomaly in the rise in consumption rates by households in the second quintile.

[77] In Appendix G, I summarize the share of disposable income and consumption held by different wealth quintiles for both the model and data. In the absence of any calibration target, the model is able to explain the rising share of disposable income and consumption in wealth. Importantly, despite holding negligible share of total wealth in the model economy (see Table 2), the bottom 40 percent of households still explain more than 20 percent of total consumer spending in the economy.

the distribution of households for explaining the response of aggregate consumption.

6.4.2 Portfolio adjustment behavior of households

A sharp fall in aggregate investment in the model economy is driven by households, as a whole, adjusting their portfolios toward safe liquid assets in a recession. In particular, while most households do not change their holdings of illiquid assets, a small fraction of wealthy households, holding most of the illiquid assets in the economy, choose to liquidate some of it. To show this, Table 15 summarizes the share of adjusting households across wealth quintiles, and Table 16 shows the share of adjustors, in each quintile, that disinvest illiquid assets in the steady-state, and in the first two periods following the combined TFP and disaster risk shocks.[78,79]

Table 15 shows that most households are not actively engaged in adjusting their holdings of illiquid assets. Only 18 percent of the total population, and 15 percent of the top 20 percent of households, actively adjust their portfolio each period in the steady-state. What is important is that, while the share of total adjustors changes little in the model recession, more wealthy adjustors reduce their illiquid asset holdings, as shown in Table 16. For instance, the fraction of adjustors disinvesting illiquid assets in the top 40 percent of households increases by 14 to 21 percentage points relative to the steady-state. These households are likely to be those who see a severe drop in expected income, and respond by increasing their liquid wealth to smooth consumption. Given that this only explains the number of adjustors and more than 95 percent of the total illiquid wealth is held by the top 40 percent of households, the actual amount of illiquid wealth monetized by the wealthiest 40 percent is significant. This drives a sharp drop in investment in the recession.

[78]This includes both working ages and retired households.

[79]It is hard to provide comparable measures in the data. Most household survey data report the market value of net wealth, making it difficult to distinguish quantity effects from price effects in the change in households' wealth.

Table 15: Share of households who actively adjust their portfolios

	steady state	impact dates	
NW Quintiles	t=0	t=1	t=2
all	0.18	0.16	0.17
Q1(poor)	0.09	0.06	0.09
Q2	0.25	0.15	0.19
Q3	0.21	0.21	0.21
Q4	0.19	0.19	0.19
Q5(wealthy)	0.15	0.15	0.15

Notes: Table 15 shows the share of households in each quintile who actively adjust their portfolios in a steady state and the first two recession dates.

Table 16: Share of adjustors who monetize illiquid assets

	steady state	impact dates	
NW Quintiles	t=0	t=1	t=2
Q1(poor)	0.0	0.08	0.08
Q2	0.28	0.36	0.26
Q3	0.33	0.63	0.51
Q4	0.52	0.65	0.67
Q5(wealthy)	0.77	0.86	0.88

Notes: Table 16 shows the share of adjustors in each quintile who disinvest illiquid assets in a steady state and the first two recession dates.

7 Concluding remarks

In this paper, I study the role of household differences, in the holdings of liquid and illiquid assets, for understanding aggregate responses in a recession associated with an increased risk of economic disaster. Solving the DSGE OLG economy with uninsurable earnings, unemployment and liquidity risk, I find that adding realistic heterogeneity – not only in the level of wealth, but also in its liquidity – is important for understanding both aggregate and household-level changes over the Great Recession. I assume that a rise in disaster risk leads households to lower their expectation of future income. The resulting negative wealth effect increases precautionary savings, in safe liquid assets, and decreases consumption. Moreover, portfolio adjustments towards safe assets lead to a fall in investment in physical capital.

Crucially, compared to a single asset model with liquid physical capital, illiquidity in wealth in a two-asset model dampens the substitution effect that arises from a fall in the expected return to high-yield but illiquid assets. Households that are not actively adjusting their holdings of illiquid assets are less responsive to changes in its return. The resulting weaker average substitution effect implies less offsetting of the negative wealth effect of a rise in aggregate risk. This drives a larger fall in aggregate consumption. Most importantly, precautionary savings motives, following an increase in the probability of a large disaster, yield powerful changes in household behavior. The movements in households' consumption and expenditure rates are qualitatively consistent with what we see in the micro-data over the Great Recession.

Though targeted, I want to emphasize that wealthy hand-to-mouth households play an unimportant role for the dynamics of aggregate consumption in a large recession. These are households that are effectively liquidity constrained as they have most of their wealth in illiquid assets. Kaplan and Violante (2014) found that such households are important for understanding the effect of fiscal stimulus because illiquidity in wealth makes these households consume a temporary tax cut instead of increasing savings. In contrast, in my paper, in a recession which involves a large persistent fall in income, wealthy hand-to-mouth households can always disinvest illiquid assets to smooth their consumption if needed. Thus, they are not the households driving the large response in aggregate consumption, instead it is the relatively wealth-poor households who increase their precautionary savings.

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A Data Appendix

A.1 Aggregate data

Real GDP, consumption and private and business fixed investment data are from BEA NIPA Tables. Consumption is defined as the total spending on non-durable goods and services. The investment includes business fixed investment, residential investment, and consumer durables. The capital stock is measured using private capital data from BEA fixed asset Tables. The total hours series are from Cociuba et al. (2018) and include civilian and military hours worked by noninstitutional population aged 16 to 64. Each series, expressed in logs, are detrended using the Hodrick-Prescott filter. Measured TFP is directly calculated using Solow Residuals consistent with the calibrated capital and labor shares in the model economy. The sample periods are from 1954Q1 to 2012Q4.

A.2 2007-2009 SCF panel data

The SCF is a household triennial data survey conducted by the Federal Reserve Board. The SCF provides detailed information on the assets and debts of households. The SCF also employs a list sample design based on the IRS data to provide a disproportionate sampling of wealthy households. Historically, the SCF had panel data from 1983 to 1989 and recently released two-year panel data for 2007 and 2009.

Net worth is defined as total assets minus total debt. Total assets include both financial and non-financial assets. Total financial assets include transaction accounts, certificates of deposit, directly held pooled investment funds, savings bonds, directly held stocks and bonds, cash value of life insurance, quasi-liquid retirement accounts, and other managed and miscellaneous financial assets. Total non-financial assets include vehicles, residential property, net equity in non-residential real estate, business equity, and other miscellaneous non-financial assets. Total debt consists of debt secured by residential property, credit card balances since last payment, installment loans, and other debts. I select households with a head who is between 25 and 85 years old and not self-employed. All the variables are deflated and expressed in 2006 dollars. Full sample weights are used.

A.3 PSID data

The PSID is a longitudinal survey of a sample of US individuals and families conducted annually from 1968 to 1977, and biennially since 1997. The original 1968 PSID sample combines the Survey Research Center (SRC) and the Survey of Economic Opportunities (SEO) samples.

For greater consistency with the SCF, I select households with a head who is between 25 and 85 years old and drop the self-employed. If income is top-coded, I multiply it by a factor of 1.5 of the top-coded thresholds following Katz et al. (1999).

The PSID provides disaggregated data on wealth since 2003, consisting of business and farm equity, transaction accounts, equity in real estate and vehicles, stocks, bonds, IRA, and debt. Table A1 summarizes averages in the SCF and PSID. Table A2 compares the distribution of wealth in the PSID to that in the SCF.

Table A1: Summary statistics of data

Data	SCF		PSID	
Year	2007	2009	2007	2009
Sample size	13,085	13,085	6,998	7,696
Age of head	49	51	49	49
Total wealth	395,539	304,427	388,513	272,066
family taxable income	75,295	67,383	75,923	81,984
consumption expenditure	-	-	36,994	34,237

Notes: All variables are expressed in 2006 dollars. For the PSID, I drop three samples with wealth less than negative 99 million dollars. (Sources: 2007-2009 SCF and PSID)

Table A2: Distribution of wealth

Year	Gini	top 1%	5%	10%	50%	90%	≤ 0
2007 SCF	0.78	29.1	52.3	64.3	96.8	100	10.3
2007 PSID	0.80	27.0	51.8	66.8	98.8	101	17.2

Notes: Table A2 shows the wealth Gini coefficient, the share of wealth held by the top 1, 5, 10, 50 and 90 wealthiest households, and the share of households with zero or negative asset holdings in the U.S. economy. For the PSID, I drop three samples with wealth less than negative 99 million dollars.

The recent waves of the PSID provide data on households' consumption. In addition to food and housing, the PSID included items on transportation, health care, education, utilities, and childcare since 1999. In 2005, additional items such as household furnishing and equipment, clothing and apparel, trips and vacations, and recreation and entertainment were added.

I construct total expenditure in the PSID as the sum of the total spending on nondurable goods and services using sample weights.[80] The PSID measures total spending on each item for the family. Since each item has a different reporting time unit, I adjust it to an annual measure. The reporting time unit varies by samples for food delivered, food eaten out, and food at home. Approximately half of the respondents reported weekly while the rest reported monthly or biweekly. I measure annual spending on these items based on individuals' reported time units. Following Krueger et al. (2016), I impute the amount of rental services from homeowners by multiplying the value of the principal residence by 4 percent. Imputed rent and property taxes are included in expenditure on housing to be consistent with the BEA measure.[81]

Table A3 summarizes the annualized average expenditure for each spending category in the PSID.[82] In Table A4 and A5, I also compare the composition of total expenditure in the PSID to that in the BEA to comprehend the representativeness of the former micro data for macro aggregates.[83] I use NIPA Table 2.3.6 Real Personal Consumption Expenditures in chain-weighted 2009 dollars (seasonally adjusted).[84] Since the BEA measures current year consumption while the PSID reporting time unit varies by item, I take the average of the current and preceding years of the expenditure in the BEA to make the time unit closer to the expenditure measured in the PSID. Moreover, I divide total expenditure in the PSID by total family size and multiply the consumption per capita by the population

[80] Durable goods include motor vehicles and parts, furnishings and equipment, recreational goods and vehicles, and other durable goods.

[81] Given that reported income is earned in the preceding year, there may exist time inconsistency between consumption and income.

[82] These numbers are broadly consistent with the 2003 estimates reported in Charles et al. (2007).

[83] Total spending in the recent PSID is comparable to that in the CEX. Indeed, Charles et al. (2007) find that the 2003 PSID covers 72 percent of the total expenditure measured in CEX.

[84] I express PSID variables in 2009 dollars for Table A3 and A4 to make them comparable to BEA variables in chain-weighted 2009 dollars.

size to make household consumption in the PSID comparable to Personal Consumption Expenditure in BEA. Table A4 shows that the PSID aggregate consumption accounts for around 50 percent of total expenditure in the BEA. In Table A5, spending on each item as a share of total expenditure in the PSID is broadly comparable to that in the BEA. This suggests that, while micro data capture less expenditure than aggregate data in total, the dynamics of consumption at the individual level may be explained by micro data.

Table A3: Annualized average expenditure for each category in the PSID

Year	2005	2007	2009	2011
Food	7,375	7,348	6,756	6,389
At home	4,920	4,929	4,672	4,447
Away from home	2,308	2,276	1,990	1,845
Delivered	147	143	94	97
Clothing	1,553	1,543	1,299	1,067
Housing	16,321	17,113	16,016	14,660
Imputed rent	7,380	7,997	6,554	5,435
Rent	2,186	2,309	2,640	2,609
Insurance	537	572	527	518
Property tax	1,671	1,751	1,737	1,638
Utility (electricity, heat, water and sewer, other)	2,362	2,328	2,325	2,173
Telephone	2,185	2,156	2,233	2,287
Transportation	5,273	5,238	4,486	4,736
Lease payment	175	198	169	98
Car insurance	1,177	813	654	592
Gasoline	2,086	2,392	1,835	2,316
Repairs	1,475	1,448	1,524	1,429
Parking	53	43	55	43
Bus and train	75	83	83	92
Taxicab	35	38	31	43
Other transportation	197	223	135	123
Education	1,682	1,799	1,572	1,431
Child care	450	381	402	333
Health care	2,896	2,918	3,068	2,777
Hospital and nursing home	315	341	427	323
Doctor	617	633	674	625
Prescription	498	449	423	454
Health insurance	1,466	1,495	1,544	1,395
Recreation	956	994	914	709
Trips	1,906	1,921	1,821	1549
Total	38,412	39,255	36,334	33,651

Notes: Sampling weights are used to calculate the average. Variables are expressed in 2009 dollars.

Table A4: Composition of total expenditure in the BEA and PSID

	2005		2007		2009	
billions of 2009 dollars	BEA	PSID	BEA	PSID	BEA	PSID
Nondurable goods	2,095	1,352	2,218	1,458	2,195	1,304
food	744	896	786	941	895	888
clothing	299	205	321	215	314	180
gasoline	299	251	297	302	284	236
other	753	n/a	814	n/a	821	n/a
Services	6,049	3,259	6,353	3,461	6,405	3,330
housing and utilities	1,753	1,663	1,832	1,858	1,871	1,702
health care	1,466	294	1,544	321	1,613	337
transportation	333	298	337	302	306	278
recreation	360	123	385	131	383	132
food services	595	n/a	629	n/a	613	n/a
financial services and insurance	686	238	731	211	728	187
other services	856	643	895	638	891	694
total	8,144	4,611	8,571	4,919	8,719	4,634

Notes: Variables in the BEA are expressed in chain-weighted 2009 dollars. Variables in the PSID are expressed in 2009 dollars. Other services include childcare, education, communication, and vehicle services.

Table A5: Spending as a fraction of total expenditure in the BEA and PSID

	2005		2007		2009	
% of total expenditure	BEA	PSID	BEA	PSID	BEA	PSID
Nondurable goods	25.7	29.3	25.9	29.6	26.5	28.1
food	9.1	19.4	9.2	19.1	10.3	19.2
clothing	3.7	4.4	3.7	4.4	3.6	3.9
gasoline	3.7	5.4	3.5	6.1	3.3	5.1
other	9.2	n/a	9.5	n/a	9.4	n/a
Services	74.3	70.7	74.1	70.4	73.5	71.9
housing and utilities	21.5	36.1	21.4	37.8	21.5	36.7
health care	18.0	6.4	18.0	6.5	18.5	7.3
transportation	4.1	6.5	3.9	6.1	3.5	6.0
recreation	4.4	2.7	4.5	2.7	4.4	2.8
food services	7.3	n/a	7.3	n/a	7.0	n/a
financial services and insurance	8.4	5.2	8.5	4.3	8.3	4.0
other services	10.5	13.9	10.4	13.0	10.2	15.0
total	100	100	100	100	100	100

Notes: Variables in the BEA are expressed in chain-weighted 2009 dollars. Variables in the PSID are expressed in 2009 dollars. Other services include child care, education, communication and vehicle services.

B Competitive investment firm

A competitive investment firm has a technology that creates capital. It holds k units of capital created last period after selling $p(z,\mu)k$ in shares to households. An investment firm rents this capital to a production firm at the rental rate $r^k(z,\mu)$ and returns the current dividend value of shares to households. It also faces convex capital adjustment cost $\Phi(k',k)$. A competitive investment firm chooses k' to maximize its profit

$$\begin{aligned} J(k, z_f, \mu) = \max_{k'} \Bigg(& (r^k(z_f,\mu) + 1 - \delta)k - (p(z_f,\mu) + d(z_f,\mu))k \\ & + p(z_f,\mu)k' - k' - \Phi(k',k) + \sum_{g=1}^{n_z} \pi_{fg} r(z_g, z_f, \mu) J(k', z_g, \mu') \Bigg) \end{aligned} \tag{4}$$

where investment firm discounts future earnings by the marginal rate of substitution of households, $r(z_g, z_f, \mu)$.

The investment firm's optimal choices satisfy the following first-order condition

$$p(z_f,\mu) - 1 - \Phi_1(k',k) + \sum_{g=1}^{n_z} \pi_{fg} r(z_g, z_f, \mu) D_1 J(k', z_g, \mu') = 0 \tag{5}$$

and the Benveniste-Scheinkman condition provides

$$D_1 J(k, z_f, \mu) = r^k(z_f,\mu) + 1 - \delta - (p(z_f,\mu) + d(z_f,\mu)) - \Phi_2(k',k). \tag{6}$$

Assuming perfect competition for an investment firm, zero profit condition, $D_1 J(k', z_g, \mu') = 0$, determines the equilibrium price of capital and dividends as follows.

$$p(z_f,\mu) = 1 + \Phi_1(k',k) \tag{7}$$

$$d(z_f,\mu) = r^k(z_f,\mu) - \delta - \Phi_1(k',k) - \Phi_2(k',k) \tag{8}$$

C Recursive equilibrium

Define the product space $\mathbf{S} = \mathbf{J} \times \mathbf{A} \times \mathbf{B} \times \{0, l_e, 1\} \times \mathbf{E} \times \{\beta_L, \beta_M, \beta_H\}$ for the distribution of households. Thus, each household is characterized by $s = (j, a, b, e, \epsilon, \beta) \in \mathbf{S}$. Given the Borel algebra $\mathcal{S}$ generated by the open subsets of $\mathbf{S}$, $\mu : \mathcal{S} \to [0, 1]$ is a probability measure over households. Households start with an initial wealth of zero and initial labor productivity drawn from $\pi^0 \sim LN(0, \sigma_\pi^2)$. At the initial period, households also draw their permanent discount factor from $\{\beta_L, \beta_M, \beta_H\}$.

A *recursive competitive equilibrium* is a set of functions

$$(v, v^a, v^n, c^a, c^n, h^a, h^n, b^a, b^n, \chi, k, n, p, d, q, w)$$

such that:

(i) (v, v^a, v^n) solves $(1) - (3)$, and (c^a, h^a, b^a) are the policy functions associated with (2) for consumption, illiquid and liquid asset savings by a household that adjusts its illiquid asset holdings. The policy functions associated with (3) are (c^n, h^n, b^n) for consumption and savings in illiquid and liquid assets by a non-adjusting household. χ is the decision rule associated with (1), where $\chi = 1$ when the fixed cost to adjust illiquid assets is paid.

(ii) The government budget is balanced

$$G_s(z, \mu) + B_s + \sum_{j=1}^{J} \sum_{e=0}^{1} \sum_{l=1}^{n_\varepsilon} \sum_{\beta=\beta_L}^{\beta_H} \int_{\mathbf{A}} \int_{\mathbf{B}} (1 - \tau_n)(s(\varepsilon_l)\mathbf{1}_{j \geq J_r} + (1 - e)\theta_u w \varepsilon_l \mathbf{1}_{j < J_r})\mu(ds)$$
$$= \tau_a d(z, \mu)k + \tau_n w(z, \mu)n + q(z, \mu)B_s^{'}$$

(iii) Markets clear

$$n(z, \mu) = \sum_{j=1}^{J} \sum_{e=0}^{1} \sum_{l=1}^{n_\varepsilon} \sum_{\beta=\beta_L}^{\beta_H} \int_{\mathbf{A}} \int_{\mathbf{B}} \varepsilon_l e \mu(ds)$$

$$k(z, \mu) = \sum_{j=1}^{J} \sum_{e=0}^{1} \sum_{l=1}^{n_\varepsilon} \sum_{\beta=\beta_L}^{\beta_H} \int_{\mathbf{A}} \int_{\mathbf{B}} a \mu(ds)$$

$$B_s(z,\mu) = \sum_{j=1}^{J}\sum_{e=0}^{1}\sum_{l=1}^{n_\varepsilon}\sum_{\beta=\beta_L}^{\beta_H}\int_{\mathbf{A}}\int_{\mathbf{B}} b\mu(ds)$$

(iv) Prices are competitively determined

$$\begin{aligned}
w(z,\mu) &= (1-\alpha)f(z)k^{\alpha}n^{-\alpha}\\
r^k(z,\mu) &= \alpha f(z)k^{\alpha-1}n^{1-\alpha}\\
p(z,\mu) &= 1+\Phi_1(G_k(z,\mu),k)\\
d(z,\mu) &= \alpha f(z)k^{\alpha-1}n^{1-\alpha} - \delta - \Phi_1(G_k(z,\mu),k) - \Phi_2(G_k(z,\mu),k)
\end{aligned}$$

where $G_k(z,\mu)$ is the aggregate law of motion for aggregate capital. Φ_1 and Φ_2 are the derivatives of Φ with respect to G_k and k, respectively.[85,86]

(v)

$$\mu'(j+1, A_0, B_0, e', \varepsilon_k, \beta) = \\ \pi_{e'}(z)\sum_{l=1}^{n_\varepsilon}\pi_{lk}\left(\int_{\Delta_1}\mu(j,da,db,e,\varepsilon_l,\beta) + \int_{\Delta_2}\mu(j,da,db,e,\varepsilon_l,\beta)\right) \quad \forall\, j$$

where $\Delta_1 = \{(a,b,e,\varepsilon_l,\beta)|h^a(j,a,b,e,\varepsilon_l,\beta;z,\mu) \in A_0,\ b^a(j,a,b,e,\varepsilon_l,\beta;z,\mu) \in B_0 \text{ and } \chi(j,a,b,e,\varepsilon_l,\beta;z,\mu) = 1\}$ and $\Delta_2 = \{(a,b,e,\varepsilon_l,\beta)|h^n(j,a,b,e,\varepsilon_l,\beta;z,\mu) \in A_0,\ b^n(j,a,b,e,\varepsilon_l,\beta;z,\mu) \in B_0 \text{ and } \chi(j,a,b,e,\varepsilon_l,\beta;z,\mu) = 0\}$.

D Equilibrium ex-dividend price and dividends

In this section, I show that the definitions of ex-dividend price and dividend are consistent with equilibrium and imply the aggregate resource constraint. Here, I simplify the model by abstracting from a government and the liquid asset market. The results can be easily generalized to the full model.

The aggregate budget constraint for all households who are adjusting illiquid wealth is

[85] Note that these equilibrium price function implies $p = 1$ and $d = \alpha f(z)k^{\alpha-1}n^{1-\alpha} - \delta$ in the steady-state.

[86] I show that the equilibrium price functions, $p(z,\mu)$ and $d(z,\mu)$, are consistent with equilibrium in Appendix D.

as follows:

$$c_a + p(z,\mu)k'_a \le w(z,\mu)n_a + (p(z,\mu) + d(z,\mu))k_a - \xi_a$$

where $x_a = \int x\mu_a$ is the sum of a variable x for all households who are adjusting.

Using definitions of ex-dividend price and dividend, above aggregate budget constraint can be re-written as:

$$c_a + (1 + \Phi_1(k',k))k'_a \le w(z,\mu)n_a + (1 + \alpha f(z)k^{\alpha-1}n^{1-\alpha} - \delta - \Phi_2(k',k))k_a - \xi_a \tag{9}$$

where k is the aggregate stock of capital.

Likewise, the aggregate budget constraint for all households who are non-adjusting illiquid wealth is:

$$c_n \le w(z,\mu)n_n \tag{10}$$

$$(1 + \Phi_1(k',k))k'_n = (1 + \alpha f(z)k^{\alpha-1}n^{1-\alpha} - \delta - \Phi_2(k',k))k_n \tag{11}$$

Imposing $x_a + x_b = x$, $w(z,\mu) = (1-\alpha)f(z)k^{\alpha}n^{-\alpha}$ and $\Phi_1(k',k)k' + \Phi_2(k',k)k = \Phi(k',k)$, equations (9)-(11) imply the aggregate resource constraint

$$c + k' + \Phi(k',k) \le y + (1-\delta)k - \xi_a$$

E Numerical method

E.1 Steady-state

Stationary equilibria involve finite-horizon dynamic programming problems with two endogenous state variables: illiquid wealth, a, and liquid wealth, b. The state space is seven-dimensional: age, illiquid wealth, liquid wealth, unemployment shock, persistent and transitory earnings shocks, and subjective discount factor. In solving the model, I combine unemployment, persistent and transitory shocks into a single idiosyncratic shock ε, decreasing two dimensions. Below, I describe the two-stage approach, which solves households' savings decisions with two assets. Here, I abstract from aggregate states for the ease of notation.

Decision rules: A two-stage approach. Below, I first describe a household problem in the first stage at age j with illiquid asset a, liquid wealth b, idiosyncratic shocks (productivity and working status) ε, and discount factor β. Let me define $v_j^m(m, a', \varepsilon_i, \beta)$ as the intermediate value over cash-on-hand, m, the future stock of illiquid wealth a', the current idiosyncratic shock ε_i, and the discount factor β. In the first stage, given a fixed cost ξ, a household chooses whether or not to adjust its portfolio. If a household adjusts its portfolio, it chooses its savings in illiquid wealth, a', using its current cash-on-hand, m. If a household chooses not to adjust its portfolio, a' becomes $\frac{(p+(1-\tau_a)d)a}{p}$.

1. First stage: Illiquid wealth problem

$$v_j\left(a, b, \varepsilon_i, \beta\right) = \max\left\{\max_{0 \le a' \le m} v_j^m\left(m - pa', a', \varepsilon_i, \beta\right), v_j^m\left(x_i + b, \frac{(p + (1-\tau_a)d)a}{p}, \varepsilon_i, \beta\right)\right\} \tag{12}$$

subject to

$$m = x_i(j, \varepsilon) + \left(p + (1-\tau_a)d\right)a + b - \xi(\kappa, a)$$

Note that if a household chooses to adjust its illiquid wealth to a', a household is also able to cash in its current stock of illiquid wealth, a, and the remaining cash-on-hand for consumption and liquid wealth in the second stage is $m - pa'$. If a household does not pay its fixed cost, it cannot adjust the current stock of illiquid wealth and the cash available for liquid wealth and consumption is the sum of labor income if working (or pension benefit if retired) and the current stock of liquid wealth, $x_i + b$.

2. Second stage: Consumption and liquid wealth problem.

In the first stage, a household decides its illiquid wealth for the next period, a'. In the second stage, given this a' and remaining cash-on-hand m, a household solves the problem below.

$$v_j^m(m, a', \varepsilon_i, \beta) = \max_{b'} \Big(u(c) + \beta v_{j+1}^0(a', b', \varepsilon_i, \beta) \Big) \tag{13}$$

subject to

$$c + qb' \leq m$$

where v_{j+1}^0 represents the expected value of a household at the beginning of the next period.

When the aggregate supply of liquid wealth is calibrated to match a rate of return on liquid wealth, solving for stationary equilibria involves three prices (w, p, d). Note that, in the absence of elastic labor supply, wages are determined by the beginning-of-period aggregate stock of capital, K. Moreover, p is fixed to one and d is the marginal product of capital net of depreciation in a steady-state as there is no aggregate capital adjustment cost involved (see Section C for the definition of equilibrium prices). Thus, the beginning-of-period aggregate stock of capital, K, determines w and d in a steady state.[87] Given the initial guess of prices, I compute decision rules and the distribution of households. The latter is determined using a large grid over age, idiosyncratic shock, illiquid and liquid wealth. I use bilinear interpolation to place decision rules onto this grid. I update prices by bisecting for the aggregate capital, iterating through the above steps, until prices converge.

E.2 Aggregate dynamics

This paper applies the backward induction method of Reiter (2002, 2010) to solve a dynamic stochastic OLG economy involving distributions defined over 60 different cohorts

[87] This is not the case in a dynamic stochastic equilibrium as K' can be different from K, involving the aggregate capital adjustment cost.

and two assets.[88] This method directly solves for the cross-sectional distributions – *proxy distribution* – over aggregate and approximate aggregate states without imposing any restriction on the shape of distributions. Thus, the method potentially allows the distribution of households to vary in rich ways following aggregate shocks.

Specifically, the method selects a *proxy distribution*, $p^{\mu}(j, a, b, \varepsilon, \beta; z, m)$, based on *distribution selection function* (DSF), which maps aggregate and approximate aggregate states to cross-sectional distributions. A DSF selects the proxy distribution that minimizes the distance to the reference distribution, $r^{\mu}(j, a, b, \varepsilon, \beta; z, m)$, subject to moment consistency conditions.[89] As shown below, solving for the DSF entails solving a large system of linear equations. With proxy distributions solved, backward induction simultaneously solves for households' decision rules and an end-of-period distribution, implying a future approximate aggregate state consistent with households' expectations. This enforces consistency between individual behavior and the aggregate law of motion in each step.[90] Lastly, I simulate the model economy and weight simulated distributions using an inverse quadratic to update the reference distributions, and thus the DSF.

The distribution of households over age and idiosyncratic shocks, as well as two assets, significantly increases the dimension of the system of equations in DSF, making it intractable. To mitigate this problem, I solve a DSF on *reduced reference distributions*. Specifically, I aggregate full reference distributions over age and idiosyncratic shocks into a small subset of age and idiosyncratic type groups and calculate the weights mapping full distributions to reduced ones to solve the *reduced proxy distributions*.[91]

I outline the detail of the algorithm as follows:

(1). Approximate the cross-sectional distribution in the aggregate state, $z = \{z_1, ..., z_{n_z}\}$,

[88] The backward induction method is different from the first-order perturbation method of Reiter (2009).

[89] For the first iteration, I use the steady-state distribution as a reference distribution. This is because I cannot simulate the model before solving it. After solving the model, reference distribution is updated by simulating the economy.

[90] By contrast, Krusell and Smith (1998) solve households' decision rules, assuming the aggregate law of motion, and then update the aggregate law of motion by simulating the model. This repeats until consistency between individual decision rule and aggregate law of motion is achieved.

[91] These weights keep the shape of proxy distributions over age and idiosyncratic shocks conditional on the level of wealth close to that of the original full distributions.

with a finite vector of statistics (moments) $m = \{m_1, ..., m_{n_m}\}$. Here, I assume m_i as i^{th}-moment. Determine asset grids for decision rules and distributions $A = \{a_1, .., a_{n_a}\}$ and $B = \{b_1, .., b_{n_b}\}$.

(2). Aggregate the full reference distribution, $r^{\mu}(j, a, b, \varepsilon, \beta; z, m)$, across all age groups and a small subset of idiosyncratic types $n_{\widetilde{\varepsilon}} \leq n_{\varepsilon}$. This results in the reduced reference distribution, $r_0^{\mu}(a, b, \widetilde{\varepsilon}; z, m)$ where $\widetilde{\varepsilon} \in \{\widetilde{\varepsilon}_1, ..., \widetilde{\varepsilon}_{n_{\widetilde{\varepsilon}}}\}$. Weights of the mapping between these two distributions are

$$\omega_0(j, a, b, \varepsilon, \widetilde{\varepsilon}, \beta; z, m) : r_0^{\mu}(a, b, \widetilde{\varepsilon}; z, m) \rightarrow r^{\mu}(j, a, b, \varepsilon, \beta; z, m)$$

(3). Choose a DSF which gives the proxy distribution, $p_0^{\mu}(a, b, \widetilde{\varepsilon}; z, m)$, of the reduced reference distribution. Specifically, $p_0^{\mu}(a, b, \widetilde{\varepsilon}; z, m)$ is the solution to the following problem that minimizes the distance to the reduced distribution $r_0^{\mu}(a, b, \widetilde{\varepsilon}; z, m)$ subject to type and moment consistency conditions. Thus, for each approximate aggregate state (z, m), a DSF solves:

$$\min_{\left\{p_0^{\mu}(a_i, b_k, \widetilde{\varepsilon}_l)\right\}_{i=1,k=1,l=1}^{n_a,\, n_b,\, n_{\widetilde{\varepsilon}}}} \sum_{i=1}^{n_a} \sum_{k=1}^{n_b} \sum_{l=1}^{n_{\widetilde{\varepsilon}}} \left(p_0^{\mu}(a_i, b_k, \widetilde{\varepsilon}_l) - r_0^{\mu}(a_i, b_k, \widetilde{\varepsilon}_l)\right)^2$$

subject to

$$\sum_{i=1}^{n_a} \sum_{k=1}^{n_b} p_0^{\mu}(a_i, b_k, \widetilde{\varepsilon}_l) = \sum_{i=1}^{n_a} \sum_{k=1}^{n_b} r_0^{\mu}(a_i, b_k, \widetilde{\varepsilon}_l), \;\; l = 1, \ldots, n_{\widetilde{\varepsilon}} \tag{14}$$

$$\sum_{i=1}^{n_a} \sum_{k=1}^{n_b} \sum_{l=1}^{n_{\widetilde{\varepsilon}}} p_0^{\mu}(a_i, b_k, \widetilde{\varepsilon}_l) a_i^{i_m} = m_{i_m}^a, \;\; i_m = 1, \ldots, n_m \tag{15}$$

$$\sum_{i=1}^{n_a} \sum_{k=1}^{n_b} \sum_{l=1}^{n_{\widetilde{\varepsilon}}} p_0^{\mu}(a_i, b_k, \widetilde{\varepsilon}_l) b_k^{i_m} = m_{i_m}^b, \;\; i_m = 1, \ldots, n_m \tag{16}$$

$$p_0^{\mu}(a_i, b_k, \widetilde{\varepsilon}_l) \geq 0, \ \forall \ i, k, l$$

where equation (14) represents type consistency conditions. These conditions imply that the density of each $\widetilde{\varepsilon}$ sums to its reference value. Equations (15) and (16) are moment consistency constraints for both assets. The last condition implies that probabilities should be positive.

Ignoring non-negativity constraints for probabilities, the first-order condition for $p_0^{\mu}(a_i, b_k, \widetilde{\varepsilon}_l)$, with λ_l, $\lambda_{i_m}^a$, and $\lambda_{i_m}^b$ as Lagrange multipliers for (14), (15), and (16) respectively, is

$$2(p_0^{\mu}(a_i, b_k, \widetilde{\varepsilon}_l) - r_0^{\mu}(a_i, b_k, \widetilde{\varepsilon}_l)) - \lambda_l - \sum_{i_m=1}^{n_m} \lambda_{i_m}^a a_i^{i_m} - \sum_{i_m=1}^{n_m} \lambda_{i_m}^b b_k^{i_m} = 0 \qquad (17)$$

Finally, I solve a system of $n_{\widetilde{\varepsilon}} n_a n_b + n_{\widetilde{\varepsilon}} + 2n_m$ linear equations in $(\{p_0^{\mu}(a_i, b_k, \widetilde{\varepsilon}_l)\}_{i=1,k=1,l=1}^{n_a,\, n_b,\, n_{\widetilde{\varepsilon}}}$, $\{\lambda_l\}_{l=1}^{n_{\widetilde{\varepsilon}}}, \{\lambda_{i_m}^a\}_{i_m=1}^{n_m}, \{\lambda_{i_m}^b\}_{i_m=1}^{n_m})$.

As I ignore non-negativity constraints, the resulting solution to the system of equations may have negative elements. If any of the elements of the solution are negative, I set those elements equal to zero and reduce the system to the remaining elements. I solve the reduced system iteratively until the solution has no negative elements.

(4). Using weights in (2), restore the full proxy distribution over age and idiosyncratic shocks, $p^{\mu}(j, a, b, \varepsilon, \beta; z, m)$.

(5). Simultaneously solve for households' decision rules and an intra-temporally consistent future approximate aggregate m'. Specifically, first guess the aggregate law of motion $G_k(z, m)$ for aggregate and approximate aggregate states. Given $v(J+1, a, b, \varepsilon, \beta; z, m) = 0$ and initial guess of value functions, solve for decision rules and value functions over aggregate and approximate aggregate states. Compute the full proxy distribution consistent end-of-period aggregate state m' and update $G_k(z, m)$. Iterate until $G_k(z, m)$ converges. Note that this solves for an aggregate law of motion alongside households' value functions.

(6). Given the value function solved by backward induction, simulate the model economy for T periods, then drop the first T_0 periods to develop new reference distributions. Let $\mu_t(j, a, b, \varepsilon, \beta)$ be the distribution of households over the simulation period $t = T_0 + 1, ..., T$.

I create new reference distributions as a weighted sum of μ_t, putting higher weights for distributions generating moments, m_t, closer to the vector of moments m. Define index sets that group dates for the same exogenous aggregate state, z, $I(z) = \{t|z_t = z\}$ where $z = z_1, ..., z_{n_z}$. Let $N(z)$ be the length of the vector $I(z)$. The reference distribution for each (z, m) is

$$r^{\mu}(j, a, b, \varepsilon, \beta; z, m) = \frac{1}{N(z)} \sum_{t \in I(z)} \frac{\delta_1(m, m_t)}{\delta(z, m)} \mu_t(j, a, b, \varepsilon, \beta)$$

where $\delta_1(m_0, m_1)$ is defined as the inverse of the Euclidian norm and $\delta(z, m) = \sum_{t \in I(z)} \delta_1(m, m_t)$.
(7). Iterate (2)-(6) to improve a DSF until no additional accuracy is achieved.

E.3 Accuracy

The model with aggregate shock is solved on the state space $\mathbf{S} = \mathbf{J} \times \mathbf{A} \times \mathbf{B} \times \{0, l_e, 1\} \times \mathbf{E} \times \{\beta_L, \beta_M, \beta_H\} \times \mathbf{Z} \times \mathbf{M}$. With aggregate shocks, the total number of grid points is $60 * 100 * 50 * 3 * 9 * 3 * 5 * 5 = 607.5$ million. The model is solved on a PC with two Intel Xeon CPU E5-2640 2.40GHz processors (40 logical cores) and 64GB RAM. To assess the accuracy for the approximate aggregate law of motion, I simulate the economy for $T = 1500$ periods and calculate Den Haan (2010b) error, $\epsilon_m = 100 * max_t|log\widetilde{m}_t - logm_t|$. Here, $\widetilde{m}_t$ is derived from the approximate aggregate law of motion, $G_k(z, m)$, while m_t is the actual mean of the distribution of households over the capital. Table E1 summarizes minimum, mean, and maximum Den Haan (2010b) error for different models solved. For example, the benchmark economy with disaster risk and two-assets has a maximum error of 2.4 percent and mean error of 0.5 percent, which suggests that the solution method is highly accurate.[92]

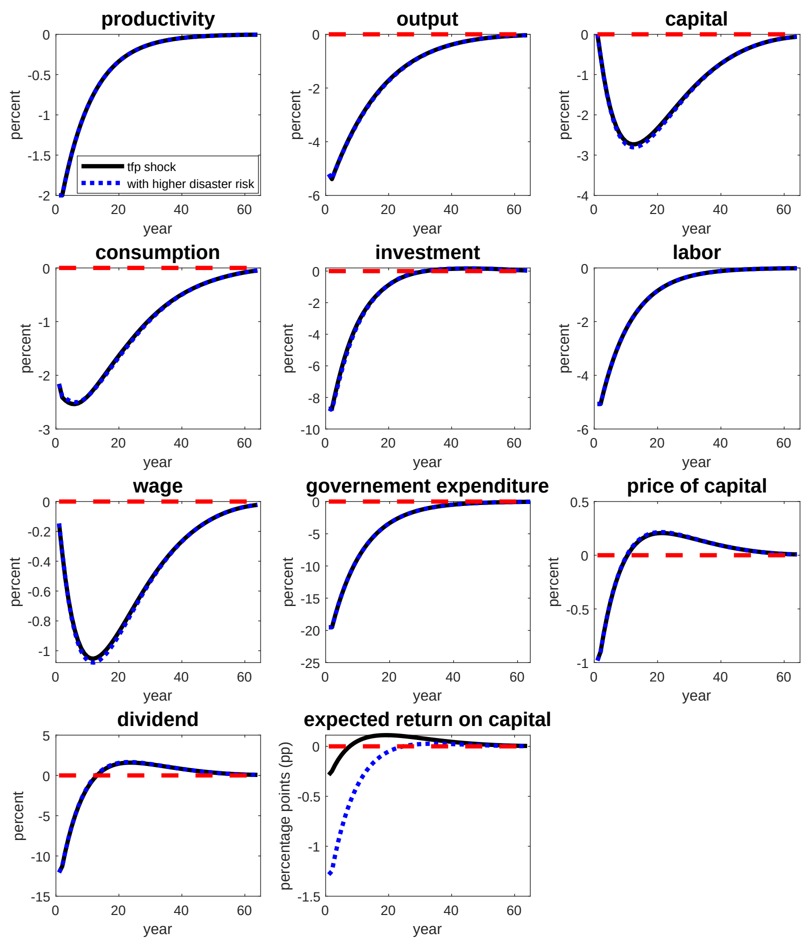
Table E1: Den Hann Error in (%)

model	min	mean	max
benchmark (r_f fixed)	0.0008	0.6619	2.2084
single asset	0.0004	0.1527	0.7782

[92] The maximum Den Hann error of 2.4 percent is similar to the accuracy achieved in Khan and Thomas (2013).

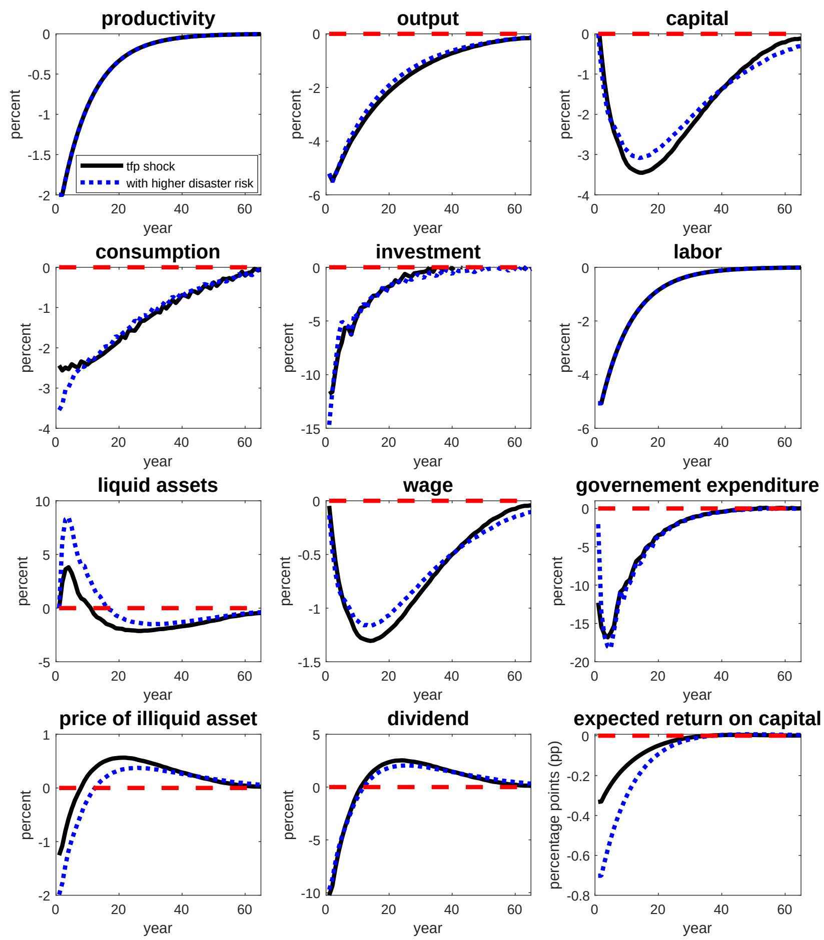
F impulse responses

Figure F1: Impulse responses in a single asset economy (Aiyagari)



Notes: Expected return on capital= $\frac{E[p'+d']}{p} - 1$. Figure F1 shows the aggregate dynamics of a single asset economy with disaster risk in response to a 2 percent TFP drop (solid black line) and to a 2 percent TFP drop alongside a rise in disaster risk (dashed blue line).

Figure F2: Impulse responses in a model with a constant return on liquid savings (r_f fixed)



Notes: Expected return on capital= $\frac{E[p'+d']}{p} - 1$. Figure F2 shows the aggregate dynamics of a fixed-return economy with both liquid and illiquid assets following a 2 percent TFP drop (solid black line) and a 2 percent TFP drop alongside a rise in disaster risk (dashed blue line). The vertical axis measures the percent changes of each variable from its simulation mean.

G Household Dynamics

Table G1: Share of total disposable income and consumption

2007	Disp. Income		Expenditure	
NW Quintiles	PSID	Model	PSID	Model
Q1(*poor*)	10.4	10.9	11.7	9.3
Q2	13.7	13.7	14.2	11.6
Q3	19.7	21.1	18.7	18.3
Q4	22.9	27.1	22.7	23.7
Q5(*wealthy*)	33.3	27.1	32.7	37.1

Notes: Table G1 shows the share of total disposable income and expenditure held by each quintile from the PSID and the benchmark economy.

Table G2: Annualized growth rates across wealth quintiles before the Great Recession

2005–2007	Avg. age		Disp. Income (%)		Expenditure (%)		Exp. Rate (pp)	
NW Quintiles	PSID	Model	PSID	Model	PSID	Model	PSID	Model
Q1(*poor*)	42	66	6.6	2.3	6.9	2.9	0.2	0.5
Q2	43	48	4.8	4.6	2.8	3.9	-1.0	-0.6
Q3	46	50	2.8	0.4	4.1	-0.2	0.7	-0.6
Q4	53	52	3.1	-2.5	3.6	-2.1	0.3	0.4
Q5(*wealthy*)	57	59	3.0	-3.7	3.3	0.6	0.2	6.9

Notes: This shows average age, annualized growth rates of average disposable income and expenditure, and percentage point changes in expenditure rates over wealth quintiles.

Table G3: Annualized growth rates across wealth quintiles during the Great Recession

2007–2011	Disp. Income (%)		Expenditure (%)		Exp. Rate (pp)	
NW Quintiles	PSID	Model	PSID	Model	PSID	Model
Q1(*poor*)	5.3	1.3	1.5	1.6	-2.1	0.2
Q2	2.8	2.5	2.5	0.8	-0.2	-1.5
Q3	1.5	-1.6	0.5	-2.6	-0.5	-1.0
Q4	0.3	-4.3	-0.6	-3.8	-0.5	0.5
Q5(*wealthy*)	-0.5	-5.4	-1.0	-0.7	-0.3	7.9

Notes: same as Table G2.

Table G4: Changes in growth rates before and during the Great Recession

	Disp. Income (%)		Expenditure (%)		Exp. Rate (pp)	
NW Quintiles	PSID	Model	PSID	Model	PSID	Model
Q1(*poor*)	-1.3	-1.0	-5.4	-1.4	-2.3	-0.3
Q2	-2.0	-2.1	-0.4	-3.1	0.9	-0.9
Q3	-1.3	-2.0	-3.6	-2.4	-1.2	-0.4
Q4	-2.8	-1.8	-4.2	-1.7	-0.7	0.1
Q5(*wealthy*)	-3.5	-1.7	-4.4	-1.3	-0.5	1.0

Notes: This shows changes in annualized growth rates of average disposable income and expenditure, and percentage point changes in expenditure rates from the PSID and the benchmark economy between normal times (2005-2007) and recession (2007-2011).

Online Appendix: Not for publication

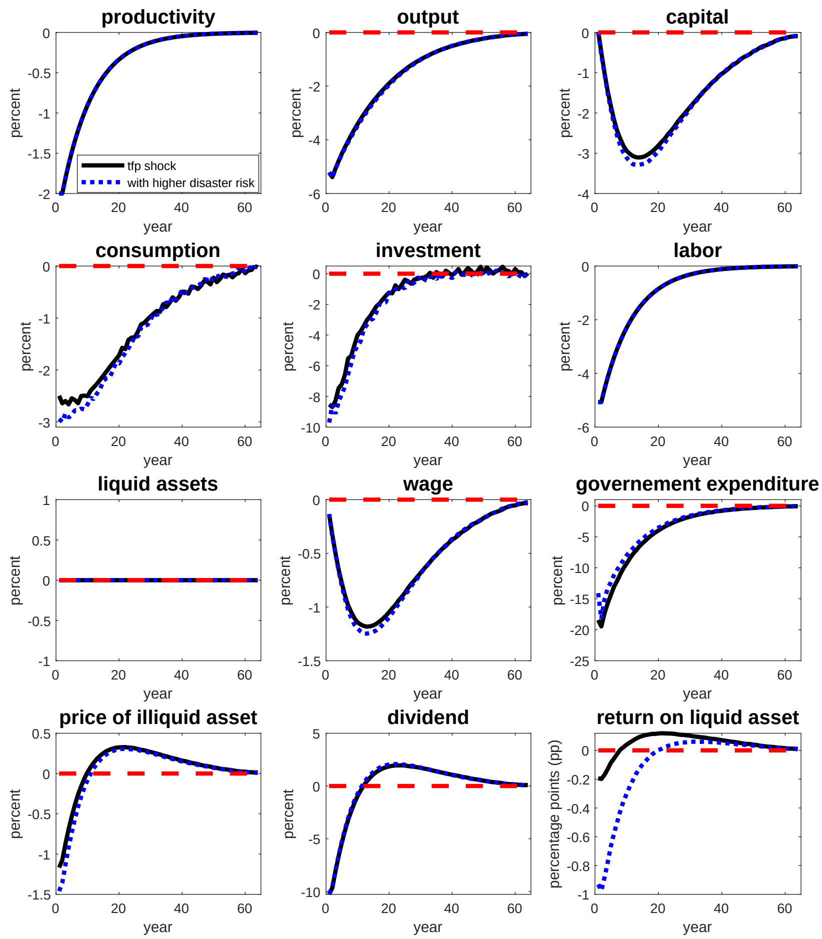
A Two-asset economy with a fixed supply of liquid assets

Figure A1 shows the impulse responses in the model economy with liquid and illiquid assets where the stock of liquid wealth is held constant. As in the benchmark economy, a higher likelihood of an economic disaster drives a large negative wealth effect as it sharply reduces households' expected income compared to a TFP shock by itself. This leads to a fall in aggregate consumption, of 3.0 percent, and a fall in aggregate investment, of 9.7 percent. This is in sharp contrast to a 2.5 percent decline of aggregate consumption and a 8.7 percent decline of investment following a persistent negative TFP shock only.

Though a rise in disaster risk explains further declines in aggregate consumption and investment, relative to a TFP shock, the additional drops in these aggregate variables are smaller in the economy with a fixed supply of liquid assets than those in the benchmark economy with a fixed return on such assets. The dampened falls in consumption and investment are in response to a fall in the return on liquid savings. Following a rise in disaster risk, households try to increase their precautionary savings through safe liquid assets. As the supply of the liquid assets is fixed, this leads to a decline in the return on liquid assets by around 1 percentage points in the equilibrium. Such a decline in the return on liquid assets gives rise to a substitution effect, which dampens precautionary savings and the fall in consumption.

Furthermore, the lower return on liquid assets discourages portfolio adjustment by households towards these safe assets. This weakens the drop in investment in physical capital. However, as shown in Krueger et al. (2016), an additional 0.5 percentage point drop in aggregate consumption is still large and significant.

Figure A1: Impulse responses in a model with a constant stock of liquid asset(B_s fixed)



Notes: Return on liquid asset $= \frac{1}{q_f} - 1$. Figure A1 shows the impulse responses of a fixed return economy with both liquid and illiquid assets following a 2 percent TFP drop (solid black line) and a 2 percent TFP drop alongside a rise in disaster risk (dashed blue line). The vertical axis measures the percent changes of each variable from its simulation mean.

B Representative agent model with two assets and diaster risk

In this section, I illustrate the implications of household heterogeneity on aggregate dynamics by solving an infinitely lived representative-agent economy with both liquid and illiquid assets and disaster risk.[93] Here, the return on liquid savings is fixed at 1 percent over time for comparability to Figure 5 of the full incomplete markets model in Section 6.2. There is no financial intermediary firm in this model. Instead, the household owns capital and receives the rental rate of capital from the production firm.

B.1 Model

Households A representative household faces convex capital adjustment cost $\Phi(k', k)$ as the financial intermediary firm in the benchmark economy. Facing convex capital adjustment cost, the household adjusts its holdings of illiquid assets smoothly in response to aggregate shocks, and this makes the solution method stable. But, it does not face non-convex adjustment cost $\xi(\kappa, a)$. To make the model comparable to the benchmark economy, I assume that the aggregate labor supply changes with aggregate shocks, $n(z)$. To be specific, at a steady state, the labor supply is, $n = 0.95$, implying a 5 percent of the unemployment rate. Following a drop in TFP by 2 percent, the labor supply decreases to $n(z) = 0.9$ to reproduce 10 percent of the unemployment rate. The household pays taxes on both labor and capital income and receives unemployment benefits θ_u proportional to unemployment $1 - n(z)$.

Defining certainty equivalent function,

$$v^0(a, b; z_f) = \left\{ \sum_{g=1}^{n_z} \pi^z_{fg} v(a, b; z_g)^{1-\gamma} \right\}^{\frac{1}{1-\gamma}},$$

the household's optimization problem is

$$v(a, b; z) = \max_{c,\, a',\, b'} \left[(1-\beta) c^{1-\sigma} + \beta v^0(a', b'; z)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{18}$$

[93] Note that this is an infinitely lived agent model without life-cycle features.

$$
\begin{aligned}
\text{s.t. } c + q(z)b' + a' &\leq b + (1 + (1-\tau_a)r(z))a + (1-\tau_n)w(z)n(z) \\
&+\theta_u(1-\tau_n)w(z)(1-n(z)) - \Phi(a', a) \\
&b' \geq \underline{b},\ a' \geq 0,\ c \geq 0
\end{aligned}
$$

Output firm maximizes its profit and the rental rate of capital and wage are determined following its optimality conditions

$$w(z) = (1-\alpha)f(z)k^{\alpha}n^{-\alpha}$$
$$r(z) = \alpha f(z)k^{\alpha-1}n^{1-\alpha} - \delta$$

Government As in the benchmark economy, the government supplies liquid assets B_s at $q(z)$. It imposes taxes on capital income at τ_a and labor income at τ_n and government revenue is used to finance unemployment benefits, interest payment on government debt, and government spending G_s. The government budget is balanced.

$$G_s + B_s + \theta_u(1-\tau_n)w(z)(1-n(z)) = \tau_a r(z)a + \tau_n w(z)n(z) + q(z)B_s'$$

A Recursive Competitive Equilibrium is a set of functions (v, c, h, b, r, w, q) such that

1. v solves (18) and (c, h, b) are the associated policy functions for consumption, illiquid and liquid asset savings.

2. Prices are competitively determined

$$w(z) = (1-\alpha)f(z)k^{\alpha}n^{-\alpha}$$
$$r(z) = \alpha f(z)k^{\alpha-1}n^{1-\alpha} - \delta$$

3. Individual decisions are consistent with aggregates

$$h(K, B, z) = G_k(K, z)$$
$$b(K, B, z) = B'$$

B.2 Parameterization

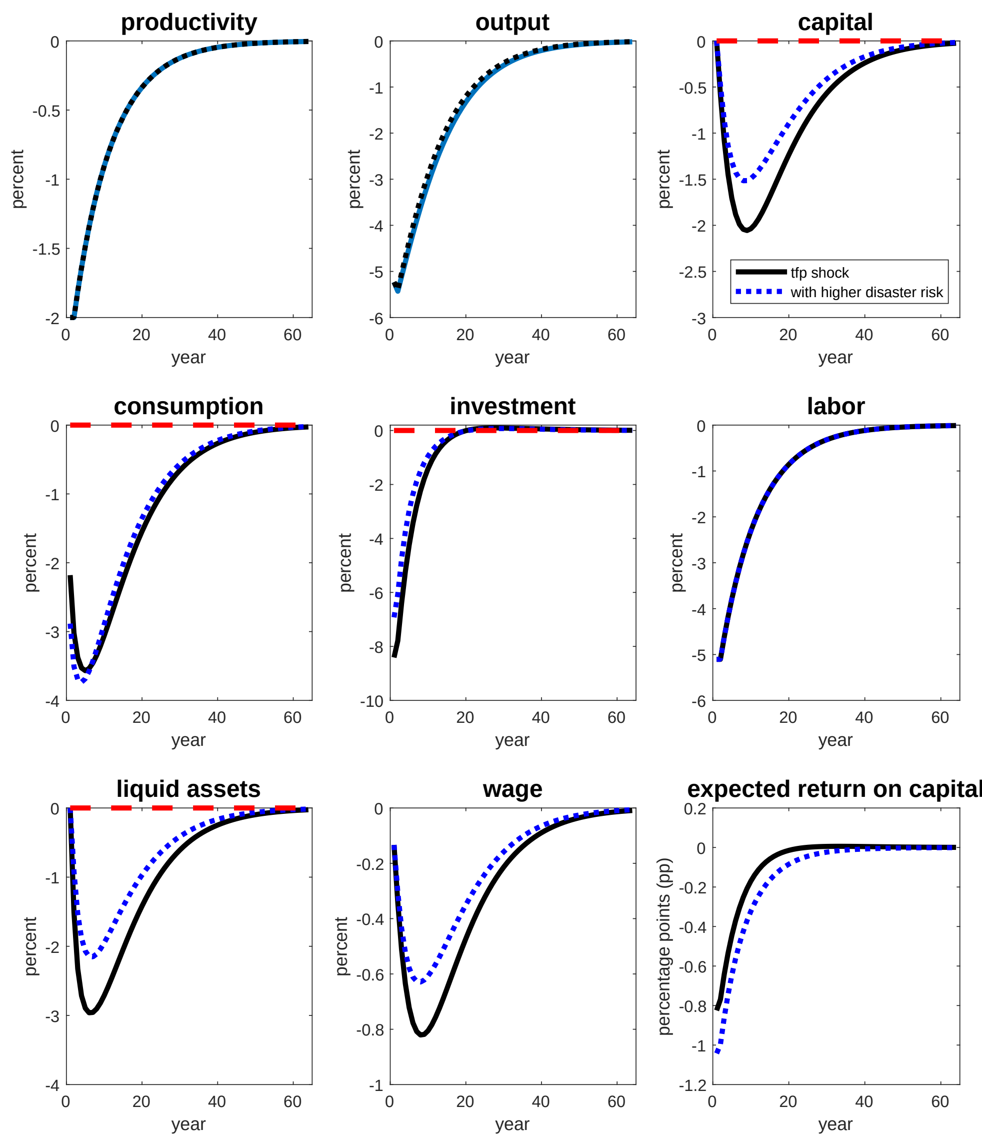
Tax rates and replacement rate of unemployment benefits are set to the values in the benchmark model. q is also set such that the return on safe asset is 1 percent consistent with benchmark economy. The values of the coefficient of relative risk aversion and IES are the same in the benchmark economy. Only β is chosen to match 2.66 of the capital to output ratio. Lastly, a household faces the same aggregate shocks in the benchmark economy.

B.3 Aggregate results

Figure B1 presents impulse responses to a 2 percent drop in TFP only (solid black lines) and those to both a TFP shock and a rise in disaster risk (dashed blue lines). As before, a higher risk of disaster leads to a large negative wealth effect. This decreases aggregate consumption and increases savings on initial impact dates compared to a TFP-shock driven recession. However, in the absence of uninsurable idiosyncratic income risk, a representative household increases its precautionary savings in both illiquid and liquid assets, and there is no flight-to-liquidity. As a result, the falls in investment and liquid assets are actually dampened when compared to a TFP-shock only recession. This is a result of capital becoming a relatively safe liquid asset in this representative-agent economy.[94] Thus, the household reallocates its increased savings across both liquid and illiquid assets. By contrast, in my benchmark economy with household heterogeneity, a rise in disaster risk further decreases consumption and investment from a TFP shock (see Figure 5).

[94] Note that, in a representative agent economy, we cannot have a discrete choice of portfolio adjustment without counterfactual volatility in aggregate investment and, therefore, GDP.

Figure B1: Impulse responses in a representative-agent economy



Notes: Figure B1 shows the aggregate dynamics of a representative-agent economy with disaster risk in response to a 2 percent TFP drop (solid black line) and to a 2 percent TFP drop alongside a rise in disaster risk (dashed blue line).

C CRRA preference

In this section, I show the aggregate responses to a rise in disaster risk when households have standard CRRA utility with the coefficient of risk aversion of 2. This implies the elasticity of substitution of 0.5. I recalibrate the economy such that it has a similar distribution of wealth as in the benchmark economy with Epstein-Zin preferences. Table C1 summarizes the moments of calibration and Table C2 describes the resulting distribution of wealth in a steady state.

Table C1: Moments targeted

moments	data	model
capital to output ratio	2.66	2.66
share of liquid asset to output	0.35	0.35
wealth Gini	0.78	0.77
fraction of households holding illiquid wealth	0.73	0.73
fraction of households with positive illiquid wealth but little liquid wealth	0.32	0.10
fraction of households holding zero or negative net worth	0.103	0.14

Source: 2007 SCF

Table C2: Distributions of net worth, illiquid assets, and liquid assets with CRRA utility

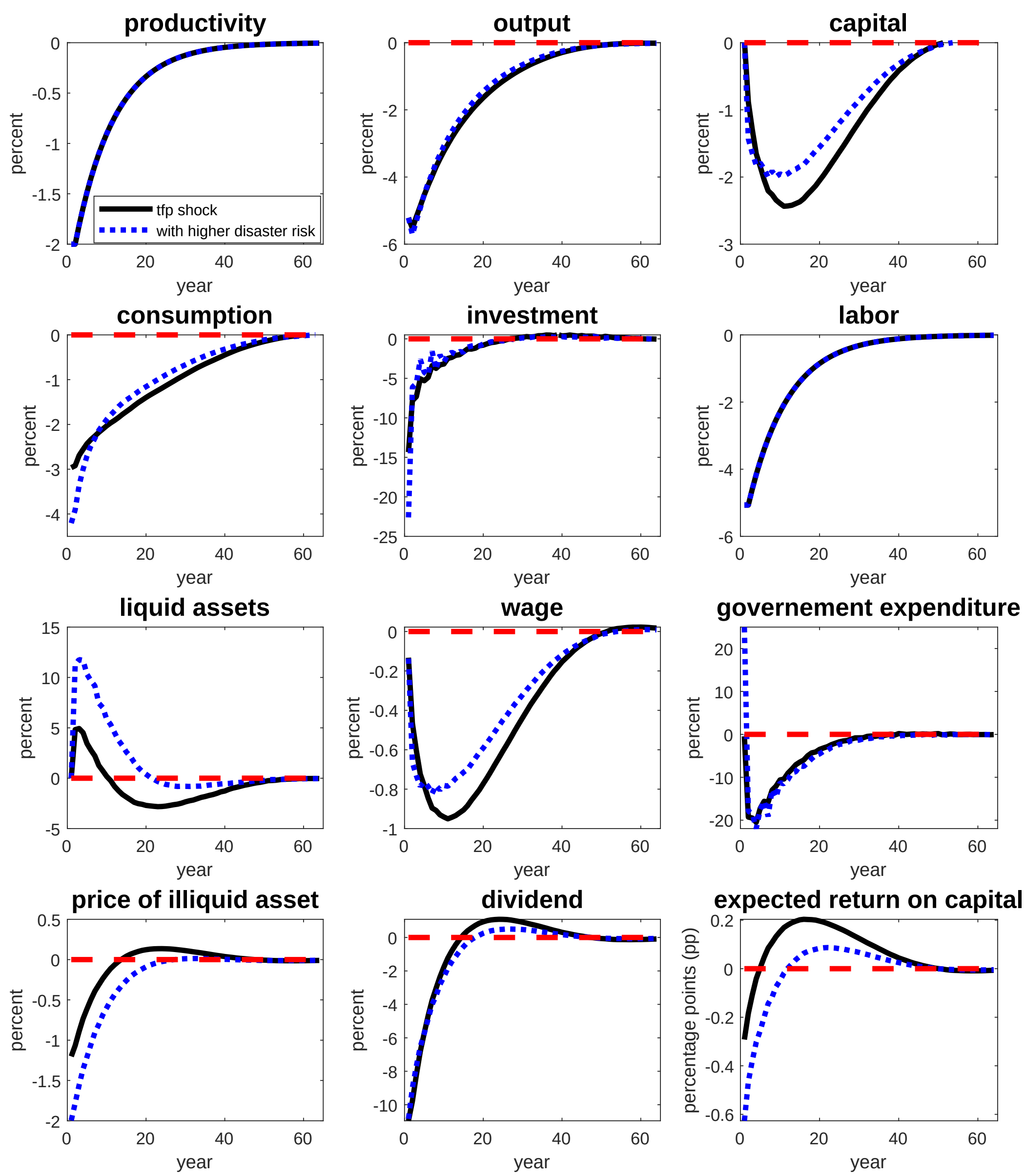
	Q1	Q2	Q3	Q4	Q5	≤ 0	Gini
Net worth	0.1	1.3	3.2	13.2	82.4	14.1	0.78
Illiquid wealth	0.0	0.5	2.5	12.7	84.3		0.79
Liquid wealth	-0.3	0.8	7.3	15.1	78.2		0.75

Notes: Table C2 shows the share of net worth, illiquid assets, and liquid assets across the wealth quintiles. It also reports the share of households with zero or negative net worth and the Gini coefficients for net worth, illiquid assets, and liquid assets in the model economy.

As seen in the fifth row of Table C1, the model with CRRA utility does not generate enough wealthy hand-to-mouth households with positive illiquid wealth but little liquid wealth. Moreover, Table C2 shows that the distribution of liquid wealth in this economy is much less skewed than that in the benchmark economy. For example, the economy with CRRA preferences only explains a liquid wealth Gini of 0.75 compared to 0.85 in the benchmark economy. Such discrepancies arise from the IES less than a unity in the model with CRRA preferences. A smaller IES implies that households' current consumption becomes less responsive to differences between the return on savings in liquid and illiquid assets. In other words, they are less willing to substitute current consumption for a higher return on illiquid assets. This leads them to hold more liquid assets as these assets are more useful for consumption smoothing. As a result, the distribution of liquid wealth is more concentrated, and it becomes hard to generate households who are willing to hold illiquid assets with little liquid wealth.

Figure C1 shows the impulse responses to aggregate shocks in the CRRA utility economy. As in the benchmark economy, a rise in disaster risk further decreases aggregate consumption and investment, compared to a TFP-shock only driven recession, and there is a strong flight-to-liquidity. Comparing this to Figure 5, we see that, in both economies, aggregate consumption falls by around 1 percentage points more following a rise in disaster risk relative to a TFP-shock only. However, the decline in aggregate investment in response to a higher probability of an economic disaster is much more pronounced with a CRRA preference compared to that with an Epstein-Zin preference. Again, this is due to the fact that households are less willing to intertemporally substitute consumption with a lower IES. Thus, when households see a higher probability of an economic disaster, they react more strongly to such a risk by increasing safe liquid assets to insure their consumption from income fluctuations.

Figure C1: Impulse responses in a model with CRRA utility (r_f fixed)



Notes: Expected return on capital= $\frac{E[p'+d']}{p} - 1$. Figure C1 shows the aggregate dynamics of an economy with CRRA preference and disaster risk in response to a 2 percent TFP drop (solid black line) and to a 2 percent TFP drop alongside a rise in disaster risk (dashed blue line).

D Business cycle moments

Table D1: Business cycle statistics in the fixed return model with disaster state

$x =$	Y	C	I	N	B	$E(r) - r_f$
mean(x)	1.154	0.748	0.200	0.688	0.322	0.079
σ_x/σ_y	(4.205)	0.476	1.974	0.645	0.974	0.077
$corr(x, y)$	1.000	0.947	0.755	0.904	-0.259	-0.111

Notes: This table shows means, relative standard deviations and contemporaneous correlation with GDP. r_f is the return on safe liquid assets and $E(r) - r_f$ is the expected liquidity premium. All series are HP-filtered with smoothing parameter 100. Standard deviation of GDP is reported in parenthesis.

Table D2: Business cycle statistics in the fixed supply model with disaster state

$x =$	Y	C	I	N	B	$E(r) - r_f$
mean(x)	1.172	0.770	0.208	0.688	0.426	0.074
σ_x/σ_y	(4.186)	0.512	1.964	0.648	n/a	0.145
$corr(x, y)$	1.000	0.983	0.984	0.902	n/a	-0.344

Same as Table D1.

Table D3: Business cycle statistics in the fixed return model without disaster state

$x =$	Y	C	I	N	B	$E(r) - r_f$
mean(x)	1.190	0.769	0.205	0.700	0.329	0.078
σ_x/σ_y	(2.800)	0.526	2.112	0.851	0.457	0.049
$corr(x, y)$	1.000	0.991	0.973	0.960	-0.481	0.946

Same as Table D1.

Table D4: Business cycle statistics in an Aiyagari model

$x =$	Y	C	I	N	B	$E(r)$
mean(x)	1.266	0.889	0.260	0.688	0.0	0.054
σ_x/σ_y	(4.187)	0.476	2.146	0.647	n/a	0.154
$corr(x, y)$	1.000	0.976	0.984	0.902	n/a	-0.389

Same as Table D1.