Determinacy and E-stability with Interest Rate Rules at the Zero Lower Bound*

Yunjong Eo[†]
Korea University

Nigel McClung[‡]
Bank of Finland

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Abstract

We evaluate and compare alternative interest rate rules, namely average inflation targeting, price level targeting, and traditional inflation targeting rules, in a standard New Keynesian model that features recurring, transient zero lower bound regimes. We use determinacy and expectational stability (E-stability) of equilibrium as the criteria for stabilization policy. We find that price level targeting policy, including nominal GDP targeting as a special case, most effectively promotes determinacy and E-stability among the policy frameworks, whereas standard inflation targeting rules are prone to indeterminacy. Average inflation targeting can induce determinacy and E-stability effectively, provided the averaging window is sufficiently long.

Keywords: Zero Lower Bound; Price level targeting; Average inflation targeting; Nominal GDP targeting; Markov-Switching; Expectations;

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[†]Yunjong Eo: Department of Economics, Korea University, Seoul 02841, South Korea; Tel: +82 2 3290 2212; Email: yunjongeo@korea.ac.kr

[‡]Nigel McClung: Research Unit, Bank of Finland. Email: nigel.mcclung@bof.fi.

1 Introduction

1.1 Motivation and Main Findings

The zero lower bound (ZLB) on interest rates has become a pervasive constraint on monetary policy in several advanced economies. In some countries, notably Japan and the U.S., the ZLB is a recurring phenomenon, and the ZLB is expected to bind more frequently in the future.[1] The ZLB on interest rates or, more generally, interest rate pegs, can destabilize expectations, and it should worry policymakers. If interest rates become constrained by the ZLB under a given interest rate rule, then the rule might induce multiple equilibria ("indeterminacy"). Some of the multiple equilibria subject the economy to extraneous, beliefs-driven volatility, and therefore they might be viewed as undesirable. In addition, it is important to understand whether agents can learn a particular equilibrium when they are assumed not to have rational expectations à la Evans and Honkapohja (2001). Under adaptive learning, the ZLB and interest rate pegs more broadly can lead to dynamically unstable inflation and inflation expectations (e.g., "deflationary spirals").[2] As a result, they are both widely associated with the non-existence of an expectationally stable ("E-stable" or "learnable") equilibrium, that is, a dynamically stable rational expectations equilibrium (REE) that could emerge from an econometric learning process involving imperfectly informed agents. When expectational stability (E-stability) conditions are not satisfied, policymakers may fail to anchor expectations under learning–even if agents' initial expectations are close to policymakers' target equilibrium.

Concerns about the recurrence of ZLB events have generated interest in alternative policy frameworks such as average inflation targeting (AIT) and price level targeting (PLT) that may mitigate the problems associated with the ZLB. For example, the Federal Reserve adopted an AIT framework in August, 2020, after conducting its own policy strategy review.

[1]Many policymakers and economists have suggested that the ZLB could bind with increasing frequency in the U.S. and other economies (e.g., see Bernanke, Kiley and Roberts (2019)). Similarly, the New York Fed's Survey of Primary Dealers for the period of 2016 to 2019 indicate that market participants placed substantial probability on the prospect of a second ZLB event in the U.S. economy.

[2]See Howitt (1992), Evans and Honkapohja (2001), Evans and Honkapohja (2003) and Evans and McGough (2018) for evidence of instability under learning with interest pegs. Evans, Guse and Honkapohja (2008), Benhabib, Evans and Honkapohja (2014), and Honkapohja and Mitra (2020), among others, document similar instabilities in nonlinear New Keynesian models with a binding ZLB.

Both AIT and PLT promise inflation in excess of the inflation target following a period of below-target inflation. In principle, these alternative policy rules serve to anchor inflation expectations since they can lead to expectations of higher future inflation when inflation is below the target at the ZLB.

The main contribution of this paper is to evaluate alternative policy rules, including AIT and price level targeting rules, when interest rates are frequently constrained by the ZLB. We ask whether these alternative policy rules help to rule out multiple equilibria when the ZLB is a recurring phenomenon. In addition, we examine whether agents can learn the equilibrium of the model under the alternative policy rules using the criterion of E-stability. More generally, policymakers should consider determinacy and E-stability desiderata for monetary policy when evaluating alternatives to the inflation targeting (IT) status quo.

We conduct our analysis using a standard New Keynesian model with recurring, transient ZLB events. Similar to Bianchi and Melosi (2017), our model features a persistent, possibly recurring two-state demand shock that follows an exogenous Markov process, and a central bank that sets interest rates with the ZLB binding when demand is low (i.e., following a contractionary demand shock). Agents are uncertain about the future path of the demand and monetary policy state, but form expectations that account for the possibility of ongoing regime changes. In this framework, recurring ZLB regimes affect the stability of expectations. Thus, we investigate whether alternative monetary policy rules can preclude multiple equilibria under rational expectations, and deflationary spirals under learning, *given* the expected duration and frequency of ZLB events.[3] Specifically, we compare three simple interest rate rules that describe three related policy strategies: (i) IT (a Taylor-type rule); (ii) AIT; and (iii) PLT (a Wicksellian rule). Note that nominal GDP targeting (NGDPT) is a special case of the PLT rules when the policy coefficients for price level and output are the same.

Our findings have important implications for stabilization policy in the recent low interest rate environment. We find that the model under a simple IT rule is prone to indeterminacy,

[3]Standard adaptive learning models can make strong, counterfactual predictions about the possibility of deflationary spirals at the ZLB. In these frameworks, agents have linear forecasting models and do not anticipate eventual escape from the ZLB. In contrast, the learning agents in our model form expectations that account for the recurrence of ZLB and active monetary policy regimes. Expectations of future active policy mitigate the potential for deflationary spirals.

although under some empirically relevant assumptions about the expected ZLB duration, simple Taylor rules can promote an E-stable REE. In contrast, the model under PLT is almost certain to admit a unique, E-stable equilibrium, as long as agents put an arbitrarily small, positive probability on the prospect of exiting the ZLB regime. The AIT rule can also promote determinacy and E-stability quite effectively, provided that the measure of average inflation is sufficiently backward looking. For all these rules, we provide real-time learning simulations that demonstrate the absence of deflationary spirals and the convergence of learning agents' expectations to rational expectations when the E-stability criterion is satisfied. These findings are also applicable to a general setting of interest rate pegs.

After a brief literature review, the paper proceeds as follows. Section 2 describes our model and the policy rules under consideration. Section 3 considers a version of the model with flexible prices, thus providing intuitive reasoning for our numerical results. Sections 4 and 5 describe our numerical results for determinacy and E-stability, and related robustness concerns. In particular, Section 5 demonstrates that concerns about indeterminacy and E-instability are substantially mitigated across policy rules if agents know the maximum duration of a ZLB event, rather than believing that ZLB events can last for an indefinite period of time. Section 6 concludes. Appendix A collects the proofs of Propositions 1-3, and Online Appendix B provides additional details about the analysis and supplementary results, including results from learning simulations.

1.2 Literature Review

A number of papers have documented indeterminacy and E-instability of REE in standard models with exogenous (or pegged) nominal interest rates. It is well-known that passive monetary policy rules, including interest pegs, permit "local indeterminacy" of the target steady state, i.e., the existence of multiple stable solution paths that converge to the steady state with positive interest rates (see Woodford (2003)).[4] Howitt (1992), Evans and Honkapohja

[4] See also Aruoba, Cuba-Borda and Schorfheide (2018), Cochrane (2017), Armenter (2018) Christiano, Eichenbaum and Johannsen (2018), Bilbiie (2022), Holden (2020), and Mertens and Ravn (2014), for more on the multiplicity of equilibria at the ZLB. We focus on "local determinacy" of the target steady state with inflation equal to the inflation target, whereas Benhabib, Schmitt-Grohé and Uribe (2001) studies "global indeterminacy", i.e., existence of two steady states: the target steady state and a low inflation steady state.

(2001), and Evans and McGough (2018) demonstrate E-instability of REE under interest rate pegs when agents are learning; and Evans, Guse and Honkapohja (2008) document E-instability of the low inflation steady state with the binding ZLB, and the possibility of deflationary spirals under learning at the ZLB. In most of the above-mentioned papers, the monetary policy regime is expected to last forever, but a determinate equilibrium may exist in models with an inflation targeting policymaker and persistent, transitory passive monetary regimes, as shown by Cho (2016) and Barthélemy and Marx (2019), and earlier considered by Davig and Leeper (2007).[5] Similarly, Mertens and Ravn (2014) and Christiano, Eichenbaum and Johannsen (2018) show when expectations are stable under adaptive learning in economies that are subject to a *one-time* transient ZLB regime; and McClung (2020) shows that Markov-switching models with an inflation targeting central bank and recurring interest rate peg regimes can admit E-stable REE if interest peg regimes are not expected to last too long.[6] Therefore, the ability of policymakers to manage expectations subject to interest peg regimes such as ZLB events depends crucially on the expected duration and frequency of the interest peg regime. This paper builds on this literature by examining the stabilization properties of alternative monetary policy strategies *given* the expected ZLB duration and frequency.

Consequently, this paper also contributes to a broad literature on alternative policy frameworks in low interest rate environments. Early works, including Svensson (2003), Eggertsson and Woodford (2003), and Auerbach and Obstfeld (2005), argue that PLT or policies that engineer temporary overshooting of the inflation target are approximately optimal strategies during liquidity traps.[7] Eo and Lie (2022) compare the welfare performance of various interest rate rules such as IT, PLT, NGDPT, and AIT under the ZLB constraint and find that NGDPT is the most desirable of the policy rules. Kiley and Roberts (2017) and Bernanke,

[5]A number of papers show how to restore determinacy under persistent interest rate pegs, including Acharya and Dogra (2020), Bilbiie (2022), Gabaix (2020), Diba and Loisel (forthcoming), and Roulleau-Pasdeloup (2020).

[6]McClung (2020) and this paper focus on REE. Self-confirming restricted perceptions equilibria of models with regime-switching Taylor rules are studied by Airaudo and Hajdini (forthcoming) and Ozden and Wouters (2020) .

[7]Woodford (2003) and Giannoni (2014) explore optimal monetary policy and PLT when interest rates are not constrained by the ZLB. Similarly, Nessén and Vestin (2005) and Eo and Lie (2020) examine the welfare implications of AIT as a makeup policy. Policymakers have drawn considerable attention to these alternative policy frameworks and related academic findings (e.g., see Evans (2012), Williams (2017), Bernanke (2017)).

Kiley and Roberts (2019) study the stabilization properties of lower-for-longer strategies, and Nakata and Schmidt (2019), Nakata and Schmidt (2020), and Bianchi, Melosi and Rottner (2020) focus on pathologies of and policy strategies for recurring ZLB events. Other recent works focus on AIT, including Mertens and Williams (2019), Budianto, Nakata and Schmidt (2020), and Amano et al. (2020).

A strand of the literature studies the stabilization properties of these alternative policy frameworks when agents are capable of learning. Our study is most closely related to that of Honkapohja and Mitra (2020), who compare PLT with IT in a liquidity trap with adaptive learning agents. They find that employing PLT can guide the economy out of the ZLB, even if agents initially put little weight on information about the price level target path when forecasting inflation. In contrast, we study an environment with recurring ZLB events, and we find that deflationary spirals are absent under PLT if learning agents put a small probability on exiting the low demand state.[8]

A few recent works specifically examine PLT and determinacy at the ZLB. Armenter (2018) documents multiple stable solution paths in a model with an optimal PLT policy, including solutions for which the ZLB binds indefinitely. Thus, PLT is no panacea. However, we can rule out analogous outcomes in our model by assuming occasional active monetary regimes. Our results are more in line with those of Holden (2020), who finds that PLT in a model with an occasionally binding ZLB constraint ensures a unique perfect foresight path that converges toward the intended steady state. In contrast to Holden (2020), we abstract from occasionally binding constraints, and instead focus on a framework with stochastic, exogenous regime changes in order to study the stability of expectations *given* the frequency and duration of ZLB events. Our approach also considers both sunspot and fundamental equilibria, a broader class of interest rate rules, and in some cases, adaptive learning agents. Thus, our work and that of Holden (2020) are complementary.

[8]See Mele, Molnár and Santoro (2020) for a study on PLT and learning when the central bank is rational.

2 Model and Equilibrium Concepts

Here we present a standard New Keynesian model with a discrete-valued demand shock that induces the ZLB for our analysis. This section also introduces important concepts related to equilibrium and stability of equilibrium, as well as the different models of expectations formation considered in the remainder of this paper.

2.1 Model Description

The model is a canonical log-linearized New Keynesian model, along the lines of Clarida, Gali and Gertler (1999) or Woodford (2003), augmented to include a discrete-valued demand shock that induces the ZLB, similar to Bianchi and Melosi (2017), and also Eggertsson and Woodford (2003), Bilbiie (2022), and Nakata and Schmidt (2019), among many others. Under rational expectations, the log-linearized equations that determine the inflation gap, π_t, and the output gap, y_t, are given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + v_{s,t} \tag{1}$$

$$y_t = E_t y_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}) + u_t + v_{d,t} \tag{2}$$

where $v_{s,t}$ and $v_{d,t}$ are mean-zero i.i.d shocks. The nominal interest rate i_t is determined in conjunction with a policy rule. All variables are expressed in terms of percentage deviations from the steady state. The variable u_t is a discrete-valued shock with two realized values: u_0 and u_1. The regime variable $s_t \in \{0, 1\}$ follows a first-order Markov process according to the transition matrix

$$P = \begin{bmatrix} p_{00} & 1 - p_{00} \\ 1 - p_{11} & p_{11} \end{bmatrix} \tag{3}$$

where $p_{ij} = Pr(s_{t+1} = j | s_t = i)$ for $i, j = 0, 1$. The values of u_0, u_1, p_{00}, and p_{11} are set such that the unconditional mean of u_t is zero where $u_0 \leq 0 \leq u_1$. When the low value for the demand shock u_0 is realized, inflation and aggregate demand fall, which affects the monetary policy stance, leading the nominal rate to hit the ZLB.

Because some of the policy rules under consideration here respond directly to the price level rather than inflation, it is convenient to rewrite (1) and (2) by using $\pi_t = p_t - p_{t-1}$ and $E_t\pi_{t+1} = E_t p_{t+1} - p_t$ as follows:[9]

$$p_t = \frac{\beta}{1+\beta}E_t p_{t+1} + \frac{\kappa}{1+\beta}y_t + \frac{1}{1+\beta}p_{t-1} + \frac{1}{1+\beta}v_{s,t} \tag{4}$$

$$y_t = E_t y_{t+1} - \sigma^{-1}(i_t - E_t p_{t+1} + p_t) + u_t + v_{d,t}. \tag{5}$$

The monetary authority tacitly conducts monetary policy using simple interest rate rules, i_t^*, which will be presented later, subject to a lower bound constraint,

$$i_t = \max\{i_t^*, -\bar{i}\}$$

where i_t is the nominal policy rate and $-\bar{i} < 0$.[10] We want to study how expectations behave given the recurrence of the ZLB, and so we simply assume $i_t = -\bar{i}$ in the low demand, low inflation state ($s_t = 0$), and $i_t = i_t^*$ otherwise. Under this simplifying assumption we need not worry about the values of $\bar{i}$ and u_t, which turn out to be utterly irrelevant in our determinacy and E-stability analysis.[11] In turn, this allows us to focus on the consequences of agents' beliefs about the persistence and frequency of the low inflation, low interest rate regime (i.e., agents' beliefs about p_{00} and p_{11}) for determinacy and E-stability. Thus, we have a tractable framework for analyzing the stability of expectations *given* recurring ZLB regimes.[12]

We consider three monetary policy rules, which are expressed below in terms of log-linearized variables (after incorporating the above assumptions) and the regime indicator s_t:

[9] We implicitly define p_t as the log deviation of the price level from some constant price level target, p^*. We can allow for a time-varying price level target without altering our main results.

[10] Here, i_t is a percentage deviation from the steady state. Assuming a zero inflation target, $\bar{i} = 1/\beta - 1$, gives the ZLB constraint.

[11] Our model's determinacy and E-stability conditions do not depend on the calibration of the model's regime-switching intercept term (i.e., $\bar{i}$ and u_t). The value of the intercept matters for the existence of equilibria such that $i_t^* < -\bar{i}$ if and only if $s_t = 0$ (e.g., see Ascari and Mavroeidis (2021) for a general treatment of related existence and multiplicity issues in models with occasionally binding constraints). However, existence considerations are beyond the scope of this study, which focuses on the stability of expectations *given the frequency and persistence of ZLB events*. See Online Appendices B.2-B.3 for more details.

[12] Tractability comes at the expense of ignoring the possibility that $i_t^* < -\bar{i}$ when $s_t = 1$ *given arbitrary values of* $\bar{i}$ and u_t. However, in our numerical analysis of E-stable equilibria, we typically find u_t such that $i_t^* < \bar{i}$ if and only if $s_t = 0$.

1. **Inflation targeting (Taylor rule):**

$$i_t = s_t(\phi_\pi \pi_t + \phi_y y_t) - (1 - s_t)\bar{i} \tag{6}$$

2. **Price level targeting (Wicksellian rule):**

$$i_t = s_t(\phi_p p_t + \phi_y y_t) - (1 - s_t)\bar{i} \tag{7}$$

   Note that the PLT rule coincides with a simple NGDPT rule when $\phi_y = \phi_p$. Thus, we use (7) to study closely related PLT and NGDPT targeting strategies.

3. **Average inflation targeting:**

$$i_t = s_t(\phi_{\bar{\pi}} \bar{\pi}_{t,t-m+1} + \phi_y y_t) - (1 - s_t)\bar{i} \tag{8}$$

   where $\bar{\pi}_{t,t-m+1} = \frac{1}{m}\sum_{j=0}^{m-1} \pi_{t-j}$. We consider two values of $\phi_{\bar{\pi}}$: (i) $\phi_{\bar{\pi}} = \phi_\pi$, such that the central bank targets a simple average of inflation with a target window m; and (ii) $\phi_{\bar{\pi}} = \phi_\pi m$, such that the central bank targets the unweighted sum of the m most recent inflation observations. Both interpretations assume that the policy rate depends on a long history of inflation data. However, as shown later, the second formulation is helpful for comparing outcomes under IT, AIT, and PLT.

As presented previously, s_t is the two-state exogenous Markov process driving u_t according to (3). Because i_t is the percentage deviation of the nominal interest rate from its steady state, $\bar{i}$, the nominal interest rate is equal to zero when $s_t = 0$.

For each policy rule we consider, we can express our model in the following general form

$$x_t = A(s_t)E_t x_{t+1} + B(s_t)x_{t-1} + C(s_t) + D(s_t)v_t \tag{9}$$

where x_t collects endogenous variables such as p_t, y_t, and i_t and their lagged variables depending on the model, $C(s_t)$ is a function of u_t and $\bar{i}$, and $v_t = (v_{s,t}, v_{d,t})'$.

2.2 Rational Expectations Equilibrium

For our rational expectations analysis, we assume that agents possess complete, homogeneous information of the economy and form true mathematical expectations, $E_t x_{t+1}$, conditional on complete time-t information.[13] An REE is any mean-square stable stochastic process $\{x_t\}$ that solves the model (9) under the above-mentioned assumptions.[14] *If* a unique REE of (9) exists, then it assumes the minimal state variable (MSV) form:

$$x_t = \Omega(s_t)x_{t-1} + Q(s_t)v_t + \Gamma(s_t) \tag{10}$$

Numerous methods have been developed to obtain solutions of the form (10); here we use the forward method of Cho (2016).[15] After obtaining an MSV solution (10), we assess the uniqueness of the equilibrium using the determinacy conditions in Cho (2016) and Cho (2021), after making a few modifications to account for the presence of a regime-switching intercept term (see Online Appendix B.2 for more details). Additional details about the REE of (9) are contained in Online Appendix B.1.

2.3 Adaptive Learning Framework

As an alternative to rational expectations, we consider the adaptive learning approach in the spirit of Evans and Honkapohja (2001). This approach relies on more realistic assumptions about information availability than does rational expectations. For example, adaptive learning agents are not assumed to know the full structure of the economy when forming expectations of aggregate variables that matter for their decisions related to optimal pricing for firms, and optimal savings, labor and consumption for households. These learning agents form expectations using a subjective forecasting model for the aggregate variables, often referred to as a "perceived law of motion" (PLM) for the aggregate variables, which

[13]Note that agents do not know s_{t+j} for any $j \geq 1$ at time t.

[14] Mean-square stability is the most widely used stability concept in the Markov-switching dynamic stochastic general equilibrium (DSGE) literature. Intuitively, a stochastic process is mean-square stable if it has finite first and second moments. Interested readers are referred to Farmer, Waggoner and Zha (2009), Cho (2016), and Cho (2021) for more on mean-square stability. Note that Barthélemy and Marx (2019) and Barthélemy and Marx (2017) provide conditions for the uniqueness of a boundedly stable REE.

[15] Alternative solution techniques for Markov-switching DSGE models are developed by Foerster et al. (2016), Maih (2015), and Farmer, Waggoner and Zha (2011), among others.

they estimate in real time (e.g., using the recursive least squares method). In each period, households and firms make decisions contingent on these forecasts. After the markets clear, the aggregate implications of these decisions are summarized by the Euler equation and the Phillips curve, given the learning agents' inflation and output forecasts.[16]

A primary focus of this study is to identify policy rules (6)–(8) that select an equilibrium that is "E-stable" or "stable under (adaptive) learning." Intuitively, adaptive learning agents' PLM (and therefore the *actual* learning equilibrium law of motion) may converge to the REE law of motion in real time if the E-stability conditions are satisfied. Importantly, if the learning equilibrium converges to the (mean-square stable) REE then inflation is mean-square stable in the learning equilibrium. Thus, a deflationary spiral (i.e., a situation such that $E_0\pi_t \to -\infty$ as $t \to \infty$) does not occur under learning if agents learn an E-stable REE. More generally, E-unstable REE cannot be the outcome of an adaptive learning process, and therefore E-instability is a warning signal that inflation expectations can become severely de-anchored. Consequently, we should avoid policies that do not promote E-stability.

To derive the E-stability conditions (i.e., the conditions under which an REE is locally E-stable),[17] we first assume that agents have "contemporaneous information", i.e., $(P, x_t, s_t, v_t) \in \mathcal{I}_t$ where $\mathcal{I}_t$ is agents' time-t information set.[18] We also assume that learning agents recursively estimate the coefficients, $(a(s_t), b(s_t), c(s_t))$, of the following PLM:

$$x_t = a(s_t) + b(s_t)x_{t-1} + c(s_t)v_t + \tilde{\epsilon}_t, \tag{11}$$

where $\tilde{\epsilon}_t$ is the perceived i.i.d. noises; $x_t = (\pi_t, y_t)'$ for IT; $x_t = (p_t, y_t)'$ for PLT; and $x_t = (\pi_t, \dots, \pi_{t-m+2}, y_t)'$ for AIT. Note that (11) has the same functional form as the MSV solution (10). That is, we assume agents use a correctly-specified econometric model, but

[16] In keeping with much of the adaptive learning literature, we assume that the Euler equation and the Phillips curve describe the evolution of equilibrium output and inflation given adaptive learning forecasts. However, a large body of literature studies an alternative approach in which agents' decisions, and hence economic outcomes, depend on their long-horizon expectations. See Preston (2005), Eusepi and Preston (2011), Honkapohja, Mitra and Evans (2013), and Bullard and Eusepi (2014) for more on infinite-horizon learning.

[17] We stress that in models with lagged endogenous variables, the E-stability conditions are "local" conditions in the sense that E-stability only predicts the convergence of adaptive learning beliefs to REE beliefs if the initial beliefs are in some neighborhood of the REE beliefs.

[18] Later we discuss implications of excluding x_t from $\mathcal{I}_t$.

do not know the parameter values.

Agents update their estimates recursively by observing a sequence of time-t *temporary equilibria*, i.e., a time-t solution of (9) given time-t expectations formed by forwarding (11) one period and taking expectations using information in $\mathcal{I}_t$ and the most recent estimates of $a(s_t), b(s_t), c(s_t)$. For brevity, we summarize the determination of temporary equilibrium and process of learning in greater detail in Online Appendix B.3.[19] That appendix also establishes that learning agents' beliefs about the coefficients in (11) only converge to self-confirming values if the agents learn the coefficients of an MSV REE (10). Formally, we say that adaptive learning agents learn the REE solution (10) if $(a(s_t)_t, b(s_t)_t, c(s_t)_t) \rightarrow (\Gamma(s_t), \Omega(s_t), Q(s_t))$ as $t \rightarrow \infty$ where $(a(s_t)_t, b(s_t)_t, c(s_t)_t)$ denotes agents' estimate of $(a(s_t), b(s_t), c(s_t))$ using information available in time t. Proposition 1 of McClung (2020) derives the E-stability conditions under which agents may learn an MSV solution (10) by estimating (11) in real time and making forecasts contingent on these estimates using contemporaneous information. When the E-stability conditions fail (i.e., "E-instability" is obtained), agents will not learn the MSV solution. Interested readers are referred to Online Appendix B.3 or McClung (2020) for more details on the E-stability conditions.

2.4 The Relationship between Determinacy and E-stability

McClung (2020) also shows that determinacy implies the E-stability of the unique mean-square stable equilibrium when agents have contemporaneous information.[20] Therefore, if the model (9) is determinate, then the unique mean-square stable REE (10) is E-stable. However, the converse is not true; E-stability may select an equilibrium that the determinacy criterion would not select. Therefore, the regions of the model parameter space that generate E-stable MSV solutions are larger than those of the parameter space that generate a unique mean-square stable REE. In fact, McClung (2020) shows that the E-stability conditions for models of the form (9) can be significantly weaker than the determinacy conditions.

[19] See also Online Appendix B.4 for results from simulations of learning in our model. Online Appendix B.4 and McClung (2020) provide explicit algorithms for the recursive estimation of a PLM of the form (11).

[20] See Propositions 1 and 2 of McClung (2020) for details. Branch, Davig and McGough (2013) also study uniqueness and E-stability of (regime-dependent) REE in a class of purely forward-looking regime-switching models.

3 Analysis with a Simplified Model

Before exploring a fully fledged New Keynesian model, we first study (in)determinacy and E-(in)stability in a simplified version of the model (4)-(5). In the simplified model, as in Davig and Leeper (2007), prices are assumed to be fully flexible (i.e., as $\kappa \to \infty$ such that $y_t = 0$ for all t). However, we study a variety of targeting rules at the ZLB, while Davig and Leeper (2007) examine the equilibrium properties of recurring active and passive monetary regimes in the context of a Taylor-type rule. With flexible prices (and also assuming $v_t = 0$ for exposition), the model reduces to the Fisherian model:

$$i_t \quad = \quad E_t p_{t+1} - p_t + \sigma u_t. \tag{12}$$

To characterize the equilibrium dynamics of inflation, we need only pair the Fisher equation (12) with one of the interest rate rules from (6)–(8). Analytical determinacy results are available, which help develop our predictions and intuition for the numerical analysis presented in Sections 4 and 5.

First, consider IT (6) with the Fisher equation (12), yielding the following Markov-switching expectational difference equation for inflation:

$$\phi_\pi s_t \pi_t = E_t \pi_{t+1} + (1 - s_t)\,\bar{i} + \sigma u_t \tag{13}$$

where $\phi_\pi > 0$ and $y_t = 0$ for all t is imposed in (6) with flexible prices.

Proposition 1 considers determinacy and E-stability under IT in the simplified model with recurrent ZLB events.

Proposition 1 *Consider a simple model of inflation (13) that combines the Fisher equation* (12) *and IT* (6)*, and assume $\phi_\pi \geq 0$. Then, (13) is indeterminate and the MSV solution is E-unstable for any finite value of $\phi_\pi > 0$.*

Proof: See Appendix A.1. ■

Proposition 1 suggests that IT is always ineffective as stabilization policy in our simplified model with recurring ZLB events, regardless of their frequency and duration.[21]

[21] In the absence of ZLB events (i.e., $s_t = 1$ for all t), and assuming $\phi_\pi \geq 0$, the simplified model with

It is instructive to consider an alternative interest rate rule. Here, we consider PLT (7). If we substitute (7) into the Fisher equation (12), then we arrive at the following Markov-switching expectational difference equation for the price level, p_t:

$$p_t = (1+\phi_p s_t)^{-1} E_t p_{t+1} + (1+\phi_p s_t)^{-1} \left((1-s_t)\,\bar{i} + \sigma u_t\right). \tag{14}$$

Proposition 2 shows that we have determinacy and E-stability under PLT, provided that interest rates can respond to prices some of the time, even if policy is only unconstrained on an extremely infrequent basis.

Proposition 2 *Consider a simple model of inflation (14) that combines the Fisher equation (12) and PLT (7), and assume $\phi_p \geq 0$ and $p_{00} + p_{11} > 1$. Then, (14) is determinate and the unique REE is E-stable if and only if $\phi_p > 0$ and $p_{00} < 1$.*

Proof: See Appendix A.2. ■

According to Proposition 2, all that is required for determinacy under PLT is that the ZLB regime must be transitory ($p_{00} < 1$) and monetary policy must be expected to respond to the price level following exit from the ZLB regime ($\phi_p > 0$).[22] Importantly, there are no restrictions on p_{11}, apart from $p_{00} + p_{11} > 1$. Thus, PLT in a simplified model leads to determinacy even if the ZLB can be recurring (i.e., $p_{11} < 1$) and occuring more frequently than the unconstrained monetary regime (i.e., $p_{11} < p_{00}$).

AIT rules of the form (8) with $\phi_{\bar{\pi}} = \phi_\pi m$ are an intermediate case between IT and PLT. For example, if we set $m = 1$, then AIT (8) collapses to IT (6). For $m \geq 1$, we can rewrite the AIT rule (8) with $\phi_{\bar{\pi}} = \phi_\pi m$ in terms of the price level, p_t (assuming $\phi_y = \bar{i} = 0$ for the

IT is a determinate model of inflation if and only if the Taylor Principle, $\phi_\pi > 1$, is satisfied. If the model is indeterminate, such that multiple rational expectations equilibria exist, there will always be a unique MSV solution. The MSV solution of the simplified model is unique owing to the lack of lagged endogenous variables in the model. Therefore, it is apparent that a zero interest rate policy or, more generally, interest rate pegs (i.e., $\phi_\pi = 0$), does not promote determinacy or E-stability in the simple Fisherian model (12) with IT.

[22] In the absence of ZLB events (i.e., $s_t = 1$ for all s_t), and assuming $\phi_p \geq 0$, the simplified model with PLT is a determinate model if and only if $\phi_p > 0$ is satisfied. Woodford (2003), Giannoni (2014), and Honkapohja and Mitra (2020) show this result in more general models. Furthermore, the unique MSV solution is E-stable if $\phi_p > 0$. Hence, a permanent interest rate peg (i.e., $\phi_p = 0$) is on the boundary of the determinacy and E-stability region of the simple PLT model's parameter space.

sake of exposition):

$$
\begin{aligned}
i_t &= \phi_{\bar{\pi}} s_t \bar{\pi}_{t,t-m+1} \\
&= \phi_{\pi} m s_t \bar{\pi}_{t,t-m+1} \\
&= \phi_{\pi} s_t \left\{ (p_t - p_{t-1}) + (p_{t-1} - p_{t-2}) + \ldots + (p_{t-m+2} - p_{t-m+1}) \right\} \\
&= \phi_{\pi} s_t \left(p_t - p_{t-m+1} \right).
\end{aligned}
$$

In the limit $m \to \infty$, we have:

$$
i_t = \phi_{\pi} s_t \left(p_t - p_0 \right)
$$

where p_0 is some arbitrary initial condition. If we normalize $p_0 = 0$, then the AIT rule with $\phi_{\bar{\pi}} = \phi_{\pi} m$ clearly becomes the PLT rule when $m \to \infty$. Thus, we expect an increase in m to help yield determinacy results as predicted in Propositions 1 and 2. Section 4 considers a fully fledged New Keynesian model and shows that this is generally the case.

4 Rational Expectations and Determinacy

This section considers the New Keynesian model described by (1) and (2), and examines the determinacy properties for each policy rule. For each policy rule, we consider a benchmark calibration that is well within the range of calibrations studied in the literature: $\beta = 0.9975$, $\kappa = 0.05$, $\sigma = 2$, $\phi_{\pi} = 2$, and $\phi_y = 0.5/4$ for IT (6) and AIT (8), and $\phi_p = 0.25$ and $\phi_y = 0.5/4$ for (7), loosely following Williams (2010). Robustness concerns related to the calibration are discussed throughout this section.[23] As mentioned in Section 2, there is no need to calibrate $\bar{i}$ or u_t. The irrelevance of u_t underscores the fact that the persistence and frequency of passive monetary spells is what matters for the stability of beliefs under rational expectations or adaptive learning.

For each rule, we consider a range of transition probability calibrations that imply a transient ($p_{00} < 1$) and recurring ($p_{11} < 1$) ZLB regime. We use the average duration of

[23] Note that the variance of the i.i.d. shocks are irrelevant for determinacy and E-stability analysis (e.g., see Cho (2021) and McClung (2020)).

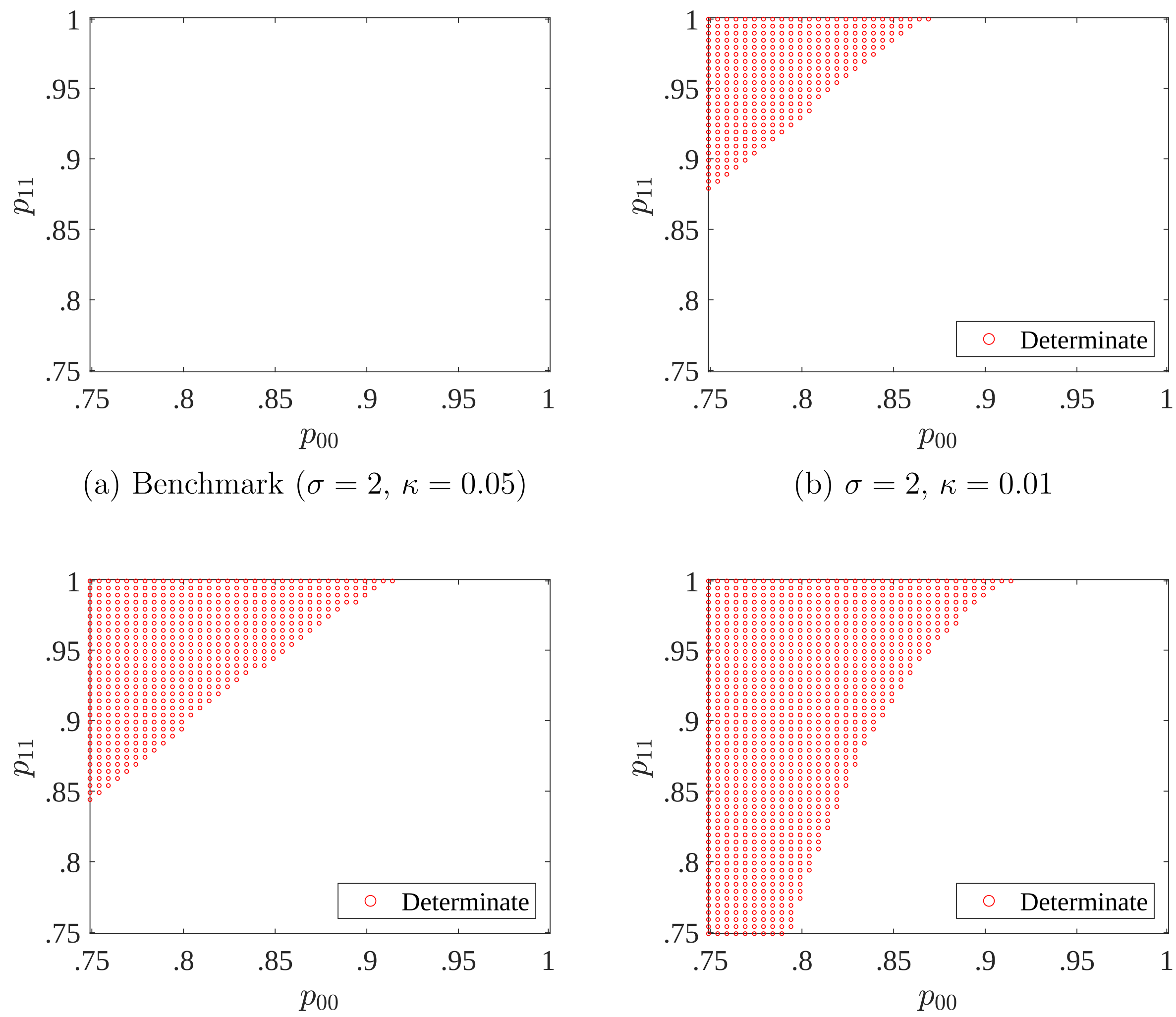
each regime, k, which is given by $(1-p_{kk})^{-1}$, for $k=0,1$, to identify reasonable values of (p_{00}, p_{11}). Recent works estimate the expected duration of the binding ZLB regime for the U.S. economy for the period of 2008 to 2015. For example, Swanson and Williams (2014) find that the Blue Chip expectation of the ZLB duration fluctuated between two and five quarters prior to the Fed's calendar-based forward guidance in 2011. After this, the expected duration increased to seven or more quarters and the median expected duration in The New York Fed's Survey of Primary Dealers increased to nine quarters.[24] Kulish, Morley and Robinson (2017) estimate the path of expected durations of the ZLB and obtain similar results, ranging from three to 12 quarters. In our model, $p_{00}=0.75$ corresponds to an expected duration of four quarters; $p_{00}=0.8$ corresponds to five quarters; $p_{00}=0.9$ corresponds to 10 quarters; $p_{00}=0.917$ corresponds to 12 quarters; and $p_{00}=0.95$ corresponds to 20 quarters.

4.1 Inflation Targeting

The baseline model with IT is formed by (1), (2) and (6). We first examine the determinacy properties of the model as a function of the transition probabilities p_{00} and p_{11}, with all other parameters set at their benchmark values. Panel (a) of Figure 1 shows that IT with recurring ZLB episodes is indeterminate for all empirically plausible values of p_{00} and p_{11} under the benchmark calibration. However, panels (b) and (c) of Figure 1 show that the determinacy region in the (p_{00}, p_{11})-space expands as the Phillips curve flattens (κ decreases) or as risk aversion increases (σ increases), holding all other parameters at the benchmark values. Intuitively, a lower κ or higher σ reduces the positive, expectations-destabilizing feedback from the expectations to the equilibrium outcomes which give rise to extraneous self-fulfilling fluctuations. A lower κ reduces the sensitivity of inflation to the output expectations because $\pi_t = \kappa \sum_{k\geq 0} \beta^k E_t y_{t+k}$; and a higher σ reduces the sensitivity of output to the real interest rate expectations because $y_t = -\sigma^{-1} \sum_{k\geq 0}(i_{t+k} - E_t \pi_{t+k+1})$. For brevity, Figure 1 does not depict the effects of reducing β. However, a lower β also reduces positive expectational feedback, thus enlarging the determinacy regions. Furthermore, aggressive active monetary policy (i.e., higher ϕ_π) can enlarge the determinacy regions when κ is sufficiently low and σ

[24] The forecast horizon in the Blue Chip consensus expectation of the ZLB duration is only six quarters. See Swanson and Williams (2014) for more.

Figure 1: Determinacy and IT (the Taylor rule)



(a) Benchmark ($\sigma = 2$, $\kappa = 0.05$)

(b) $\sigma = 2$, $\kappa = 0.01$

(c) $\sigma = 5$, $\kappa = 0.01$

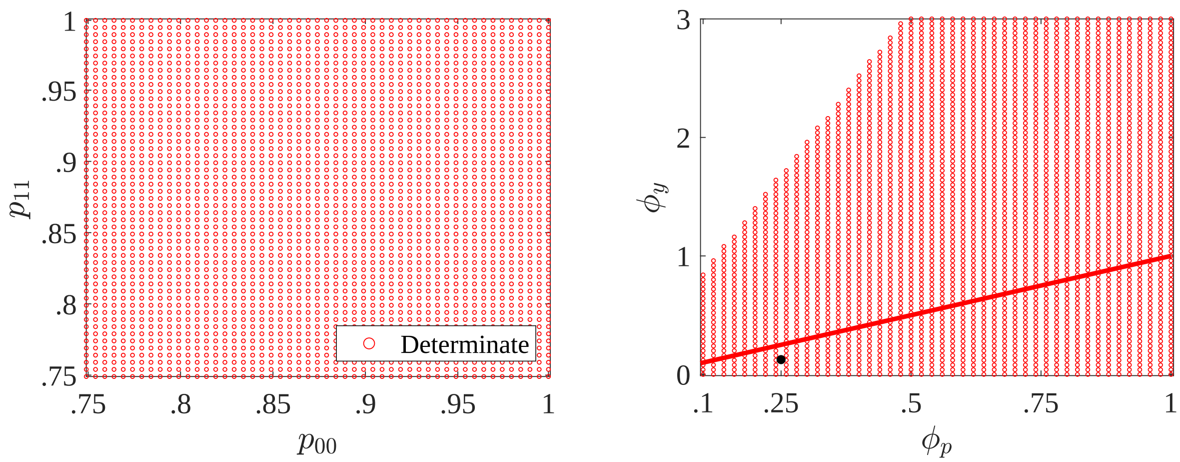
(d) $\phi_\pi = 5$, $\sigma = 5$, $\kappa = 0.01$

Note: The REE for IT rules is depicted with respect to p_{00} and p_{11} as follows. The red (circle) area denotes determinacy; and the white area denotes indeterminacy. Other model parameters are given by $\beta = 0.9975$, $\phi_\pi = 2$, and $\phi_y = 0.5/4$ throughout this exercise unless noted otherwise.

is sufficiently high (see panel (d) of Figure 1). Finally, panels (b)–(d) of Figure 1 show that the low frequency (a high value of p_{11}) and short duration (a low value of p_{00}) of ZLB events are key to ensuring a unique REE.

We conclude that IT under a Taylor-type rule is prone to indeterminacy in a model subject to recurring ZLB events. However, this problem of indeterminacy is mitigated provided the Phillips curve is sufficiently flat, monetary policy is very active away from the ZLB, or agents

Figure 2: Determinacy and PLT



(a) Benchmark ($\phi_p = 0.25$, $\phi_y = 0.5/4$) (b) $p_{00} = 0.965$, $p_{11} = 0.975$

Note: The REE for a PLT rule is depicted with respect to p_{00} and p_{11} in panel (a) and with respect to ϕ_p and ϕ_y in panel (b) as follows. The red (circle) area denotes determinacy; the white area denotes indeterminacy. The black square in panel (b) depicts the benchmark calibration, and the red line is the set of points satisfying the NGDPT restriction $\phi_p = \phi_y$. The model parameters are set at the benchmark values throughout this exercise unless noted otherwise.

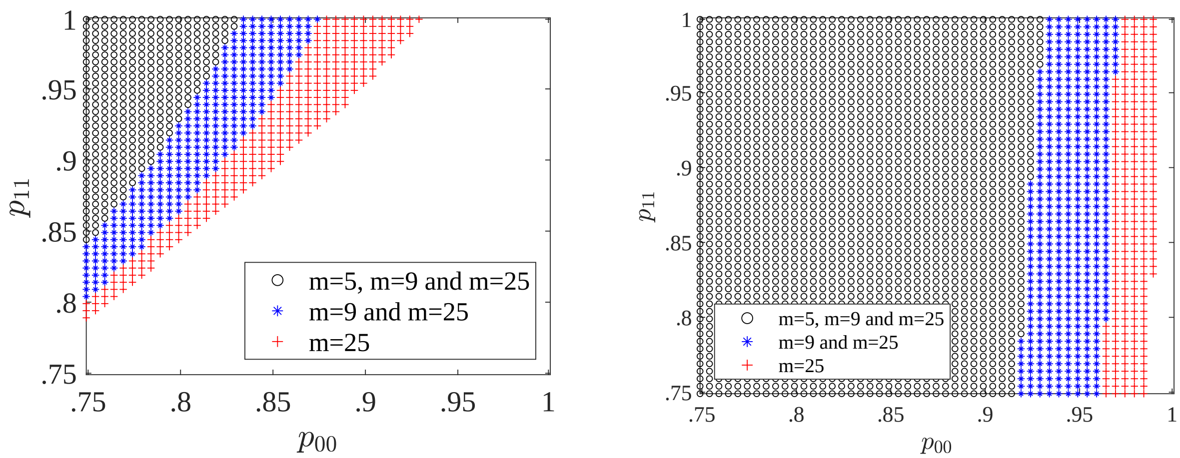
are very risk averse.

4.2 Price level targeting

Now we consider determinacy under a PLT rule of the form (7). The result is stark, and consistent with the findings from the simple Fisherian model with PLT (i.e., Proposition 2): for sufficiently small ϕ_y, we have determinacy for all (p_{00}, p_{11}), provided $p_{00} < 1$. We have determinacy even if ZLB events are more frequent and persistent than are the unconstrained policy regimes (i.e., $p_{11} < p_{00}$). Figure 2 (a) illustrates the basic result for the benchmark calibration. Furthermore, in line with Proposition 2, we could instead set $\phi_p = 0.0001$ and $\phi_y = 0$ and also obtain the same determinacy region as depicted in Figure 2 (a). Finally, when $\phi_p = \phi_y$ such that the PLT rule (7) implements nominal GDP targeting policy, we again find the determinacy region depicted in Figure 2(a).[25] Therefore, NGDPT policy is a highly

[25] We explored determinacy regions in the model with many values of $\phi_p \in [0.0001, 100]$ given small ψ_y and also $\phi_p = \phi_y \in [0.0001, 100]$. We invariably obtain the determinacy region depicted in Figure 2(a).

Figure 3: Determinacy and AIT



(a) Simple Average Target ($\phi_{\bar{\pi}} = \phi_\pi$). (b) Unweighted Target ($\phi_{\bar{\pi}} = \phi_\pi m$).

Note: The REE for AIT with various target windows of m is depicted with respect to p_{00} and p_{11}. The black (circle) region is the determinacy region for $m = 5$; the determinacy region for $m = 9$ consists of the black and blue (asterisk) regions; the determinacy region for $m = 25$ consists of the black, blue, and red (plus) regions; and the white region denotes indeterminacy. The model parameters are set at the benchmark values throughout this exercise unless noted otherwise.

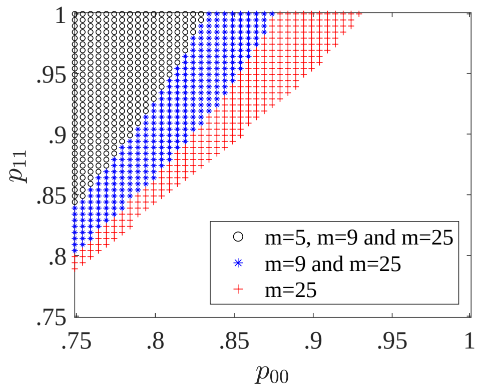
effective means of stabilizing expectations when the ZLB is expected to bind frequently.

Proposition 2 and the numerical results presented in this section indicate that policymakers can almost always manage rational expectations under PLT or NGDPT in an economy subject to ongoing ZLB events. However, there is a caveat: policymakers may destabilize the economy when responding too aggressively to output relative to the price level in (7). Figure 2 (b) shows that large values of ϕ_y (relative to ϕ_p) may lead to indeterminacy for a calibration of p_{00} that matches the persistence of the U.S. ZLB episode of 2008 to 2015. This result suggests that central banks targeting the price level should respond only mildly to output.

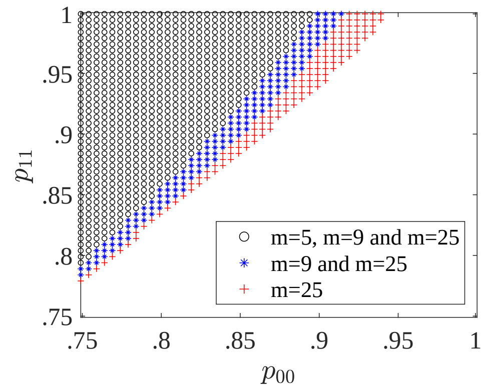
4.3 Average inflation targeting

In this section, we examine the model incorporating the AIT rule (8), the New Keynesian Phillips curve (1), and IS curve (2). We consider three measures of average inflation, indexed by $m = 5, 9, 25$, that vary in the degree of history-dependence. These values of m correspond

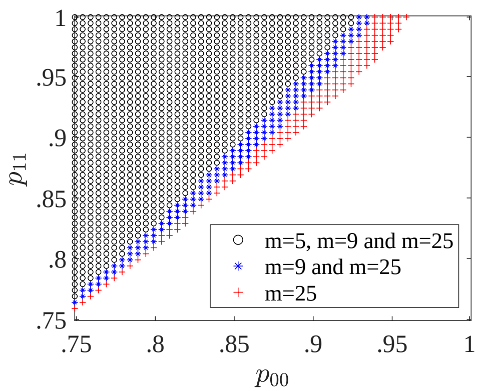
Figure 4: Determinacy and AIT under Alternative Parameterizations



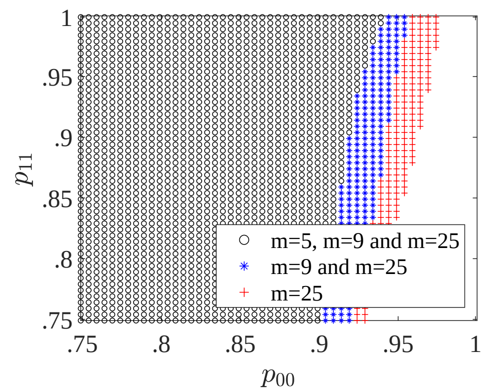
(a) Benchmark ($\sigma = 2$, $\kappa = 0.05$).



(b) $\sigma = 2$, $\kappa = 0.01$.



(c) $\sigma = 5$, $\kappa = 0.01$.



(c) $\sigma = 5$, $\kappa = 0.01$, $\phi_{\bar{\pi}} = 5$.

Note: For various parameterizations, the REE for an AIT rule is depicted with respect to p_{00} and p_{11} as follows. The black (circle) region is the determinacy region for $m = 5$; the determinacy region for $m = 9$ consists of the black and blue (asterisk) regions; the determinacy region for $m = 25$ consists of the black, blue, and red (plus) regions; and the white region denotes indeterminacy. This figure assumes the simple AIT rule (8) with $\phi_{\bar{\pi}} = \phi_\pi$. The model parameters are set at the benchmark values throughout this exercise unless noted otherwise.

to a target that averages over the past year, two years, and six years of inflation data, respectively. As mentioned in Section 2.1, we also consider two interpretations of the AIT rule. First, we set $\phi_{\bar{\pi}} = \phi_\pi$, such that the central bank targets a simple average of the most recent m quarters of inflation data. Second, we set $\phi_{\bar{\pi}} = \phi_\pi m$ and interpret the target as an unweighted sum of the most recent m quarters of inflation. In this policy framework, AIT with $\phi_{\bar{\pi}} = \phi_\pi m$ converges to PLT (7) as $m \to \infty$, as argued in Section 3.

As shown in Figure 3, under both interpretations of the rule, a higher m expands the determinacy region in the (p_{00}, p_{11})-space. Thus, we have the smallest determinacy region for IT (6), which is equivalent to AIT (8) with $m = 1$, and the largest determinacy region for the PLT rule (7), which can be obtained from AIT (8) as $m \to \infty$. In addition, as expected, the determinacy regions under the unweighted sum interpretation ($\phi_{\bar{\pi}} = \phi_\pi m$) are strictly larger than those under the simple average inflation target interpretation ($\phi_{\bar{\pi}} = \phi_\pi$), because the former generates a more active response to inflation during unconstrained regimes.

Figure 4 demonstrates the effects of varying κ, σ, and ϕ_π under the simple average inflation target interpretation (that is, $\phi_{\bar{\pi}} = \phi_\pi$; see Online Appendix B.5 for results corresponding to the unweighted case with $\phi_{\bar{\pi}} = \phi_\pi m$). As we found in our determinacy analysis with IT, a lower κ, higher σ, lower β, and higher ϕ_π enlarge the determinacy regions under AIT. The same intuition as before applies to the AIT case.

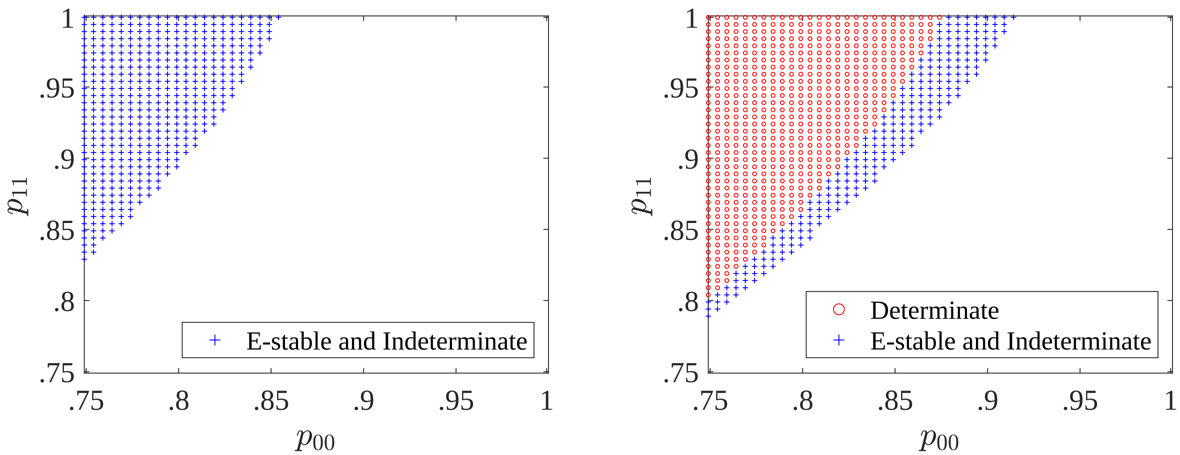
5 Adaptive Learning and E-stability

This section considers the New Keynesian model of Section 2.1, and examines the E-stability properties for each policy rule. We also demonstrate useful applications of E-stability, and consider robustness issues.

5.1 (In)Determinacy and E-(in)stability

Our discussion in Section 2.4 predicts the first basic conclusion of our E-stability analysis under the model of adaptive learning: a determinate equilibrium is E-stable. However, while determinacy implies E-stability, the converse is not true. This implies that using E-stability as a criterion instead of determinacy/indeterminacy may alter the evaluation of alternative policy rules. Therefore, we can think of the E-stability criterion as a minimal requirement for stabilization policy. We examine this possibility using our fully fledged New Keynesian model developed in Section 2.1. Figure 5 (a) shows that IT (6) is capable of generating a unique E-stable MSV solution, despite model indeterminacy over the entire parameter space of (p_{00}, p_{11}). Similarly, Figure 5 (b) shows that indeterminate models with the AIT rule (8) admit E-stable solutions. Finally, because PLT (7) generates determinacy regions that virtually exhaust the model parameter space (assuming ϕ_y is not too large), we find

Figure 5: Determinacy and E-stability



(a) IT

(b) AIT ($\phi_{\bar{\pi}} = \phi_{\pi}$)

Note: The REE for IT and AIT are depicted with respect to p_{00} and p_{11} in panels (a) and (b), respectively. The red (circle) area denotes determinacy and E-stability; the blue (plus) area denotes indeterminacy and E-stability; and the white area denotes indeterminacy and E-instability. AIT has the target window of $m = 9$ for all the simulations. PLT always leads to determinacy and E-stability for the entire parameter space considered.

that E-stability regions also exhaust the policy parameter space. In short, while IT can be associated with E-stability for some parameter region that is associated with indeterminacy, and AIT has a larger parameter region of E-stability than that for determinacy, PLT always leads to determinacy *and* E-stability for the entire parameter space considered (assuming ϕ_y is not too large). Therefore, PLT is the most effective stabilization policy according to our determinacy and E-stability analysis.[26]

[26] We also considered the following lagged information assumption: $(P, x_{t-1}, s_t, v_t) \in \mathcal{I}_t$, but $x_t \notin \mathcal{I}_t$. Our numerical E-stability results for the model (1)-(2) and any one of the policy rules from (6)-(8) are the same under this alternative lagged information assumption as those under the contemporaneous information assumption. Also note that it is quite uncommon for indeterminate models to yield an E-stable REE in standard linearized models under assumptions analogous to those in Section 2. For example, the model given by (1), (2), and (6) *without regime switching* admits an E-stable (and dynamically stable) solution if and only if the model is determinate. See Bullard and Mitra (2002) and Evans and McGough (2005) for more details.

5.2 E-stability and Deflationary Spirals

It has been well-established that deflationary spirals may occur under adaptive learning when interest rates are pegged at zero (e.g., see Evans, Guse and Honkapohja (2008)). This section illustrates how and why deflationary spirals are absent at the ZLB when agents are learning adaptively but are sufficiently optimistic about the possibility of escaping the current liquidity trap (i.e., they forecast regime changes, and p_{00} (p_{11}) is sufficiently low (high) to deliver an E-stable REE). For the purpose of this analysis, we define a deflationary spiral as

$$\lim_{t\to\infty} E_0\pi_t = -\infty.$$

The usual mathematical expectation operator E_0 denotes model-consistent expectations (which may not coincide with agents' expectations).

First, recall that the E-stability conditions can predict convergence of the learning equilibrium law of motion to the mean-square stable REE law of motion. Formally, convergence obtains in real time if $(a(s_t)_t, b(s_t)_t, c(s_t)_t) \to (\Gamma(s_t), \Omega(s_t), Q(s_t))$ for $s_t = 0, 1$ as $t \to \infty$, given $(a(s_t)_0, b(s_t)_0, c(s_t)_0)$ in some suitable neighborhood of the REE. If the learning equilibrium law of motion converges to the REE law of motion in real time then deflationary spirals will not occur because mean-square stability ensures that $lim_{t\to\infty}E_0\pi_t$ is finite (e.g., see Cho (2016) for details). Online Appendix B.4 provides results from extensive numerical simulations which demonstrate that E-stability predicts convergence of the learning equilibrium to the mean-square stable REE, and hence the absence of deflationary spirals under learning. In simulations of the E-unstable cases, the learning equilibrium invariably diverges from the REE and inflation can become dynamically unstable. The results reported in Online Appendix B.4 furthermore confirm our findings in Section 5.1; PLT dominates its alternatives with respect to anchoring expectations to rational beliefs, particularly for higher values of p_{00}.

However, E-stability may only predict convergence to rational expectations if initial beliefs are sufficiently "local" to the rational beliefs. If instead agents' initial inflation expectations are very low ("pessimistic") relative to the rational expectations then the ZLB may endogenously bind under learning in a state of the world where the REE interest rate would be positive. In such cases, the E-stability conditions may still prove useful for predicting the dynamic stability of inflation at the ZLB.

To illustrate this point, we set up a simple experiment in the model (1)-(2) paired with one of the policy rules, in which agents' initial pessimistic beliefs (i.e., *not* the fundamental shock to demand) causes the ZLB to bind at time $t = 0$. In this experiment, we shut down all fundamental shocks (i.e., $v_t = u_t = 0$) to isolate the role that regime-switching expectations play in preventing a deflationary spiral under learning. Thus, we restrict our attention to an "expectations-driven liquidity trap" which occurs because expectations are initially unanchored from rational expectations and not because of some fundamental shock to the economy.[27] Under these assumptions, the model can be cast in the form:

$$x_t = A(s_t)\hat{E}_t x_{t+1} + B(s_t)x_{t-1} + C(s_t) \tag{15}$$

where $s_t = 0$ if the ZLB binds at time-t and $s_t = 1$ otherwise. To pin down s_t without fundamental shocks we must clarify the timing of temporary equilibrium: in the beginning of t agents assume $s_t = 0$ ($s_t = 1$) when forming time-t expectations, unless they observed positive (zero) interest rates at the end of $t-1$. We use $\tilde{s}_t = s_{t-1}$ to denote the agents' subjective beliefs about s_t. The agents believe the ZLB regime has persistence $p_{00} < 1$ and the unconstrained regime has persistence $p_{11} = 1$. We assume the PLM:

$$x_t = a(\tilde{s}_t)_{t-1} + \Omega(\tilde{s}_t)x_{t-1} + \epsilon_t \tag{16}$$

where $\Omega(s_t)$ is from the REE law of motion and $a(s_t)_t$ follows a simple constant-gain learning

[27] Our notion of expectations-driven liquidity trap should not be confused with the expectations-driven liquidity traps studied by Mertens and Ravn (2014) or Nakata and Schmidt (2020) which occur as a consequence of sunspots in a REE.

process:[28]

$$a(k)_t = a(k)_{t-1} + \psi(k) R(k)_t^{-1}(x_t - \Omega(k) x_{t-1} - a(k)_{t-1}) \tag{17}$$

$$R(k)_t = R(k)_{t-1} + \psi(k)(1 - R(k)_{t-1}) \tag{18}$$

where $\psi(k) = \psi \in (0,1)$ if $s_t = k$ and 0 otherwise, and $k = 0, 1$. $R(k)_t$ is the estimate of the second moment of the regressor using data available up to and including time t. In this case, the regressor is a constant and hence $R(k)_t \to 1$ for $k = 0, 1$. We assume that ψ is constant for exposition's sake. We set $a(0)_{-1}$ to a non-zero (pessimistic) value which causes the ZLB to initially bind at $t = 0$, and we assume $a(1)_{-1}$ is the zero vector which implies that agents believe the economy will eventually return to the intended steady state with positive interest rates if the (perceived) transient ZLB event ends. Note that $a(0)_{-1} \neq 0$ is not rational since the absence of fundamental shocks implies that the economy is always in the intended steady state in the corresponding REE.[29]

After agents form time-t expectations using beliefs $a(k)_{t-1}$, equilibrium interest rates are set according to $\max\{i_t^*, -\bar{i}\}$, and this determines s_t. The temporary equilibrium is therefore:

$$x_t = \left(I - A(s_t)\hat{E}_t\Omega(\tilde{s}_{t+1})\right)^{-1}\left(A(s_t)\hat{E}_t a(\tilde{s}_{t+1})_{t-1} + B(s_t)x_{t-1} + C(s_t)\right).$$

Note that in our setup, $s_t = \tilde{s}_t$ when the economy is at the ZLB and in all periods after lift-off from the ZLB. Only in the period of lift-off we will have $\tilde{s}_t = 0$ and $s_t = 1$. Under these assumptions we ask: will a deflationary spiral take place in real time?

Through substitution we can reduce the law of motion for $a(0)_t$ during the ZLB regime

[28] Conventional adaptive learning models assume that agents learn $\Omega(s_t)$. However, in the models we now consider, learning the intercept term is a far more demanding task for learning agents, and so we simplify the analysis and abstract from the role of initial beliefs about $\Omega(s_t)$ by assuming it is known by the agents. Without much loss of generality, one may assume the REE is the unique, mean-square stable MSV solution. See Online Appendix B.4 for examples in which agents learn about all of the coefficients of the PLM.

[29] In this section and throughout the paper, we abstract from the possibility that rational agents coordinate on the second steady state equilibrium with permanently binding ZLB.

to a simple VAR(1) process:

$$
\begin{aligned}
a(0)_t &= \left(I + \psi R(0)_t^{-1}\left(p_{00}F(0) - I\right)\right) a(0)_{t-1} + \\
&\quad \psi R(0)_t^{-1} \left((1-p_{00})F(0)a(1)_{t-1} + \left(I - A(0)\sum_{j=0}^{1} p_{0j}\Omega(j) \right)^{-1} C(0) \right) \\
F(0) &= \left(I - A(0)(p_{00}\Omega(0) + (1-p_{00})\Omega(1))\right)^{-1} A(0).
\end{aligned}
$$

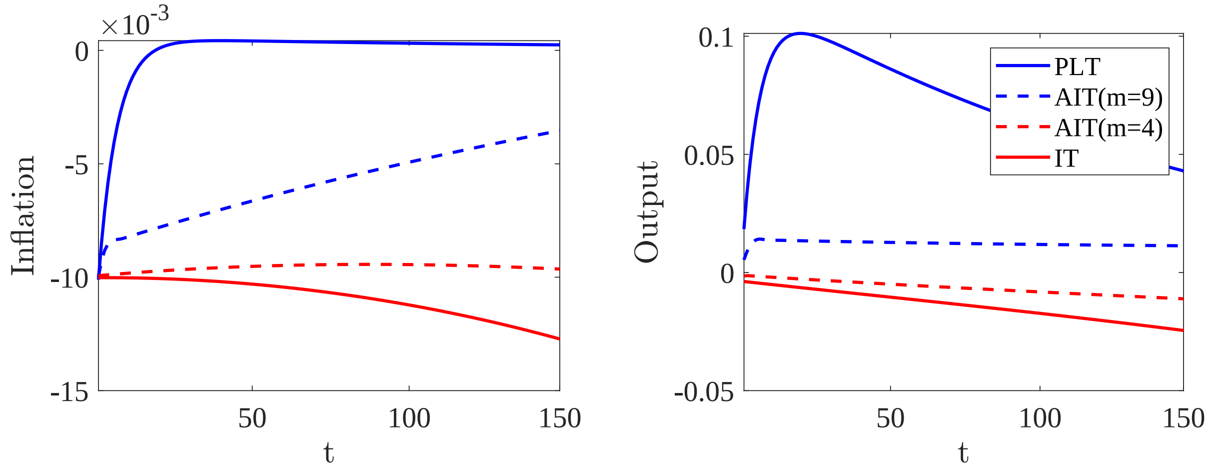
Since $R(0)_t \to 1$ almost surely and $a(1)_t = a(1)_{-1} = 0$ is fixed while agents are updating beliefs at the ZLB, $a(0)_t$ converges if the eigenvalues of $p_{00}F(0)$ are inside the unit circle. By inspecting (15)-(17) jointly, we can deduce two implications of convergence of $a(0)_t$. First, since $a(1)_t$ is fixed at the ZLB, x_t will converge if $a(0)_t$ converges. Second, x_t can converge towards levels of inflation and output for which the ZLB no longer binds if agents have sufficiently high expectations for long-run levels of inflation and output in the unconstrained monetary regime (e.g., if $a(1)_{-1}$ is the zero vector). Thus, the economy avoids a deflationary spiral and can escape the expectations-driven liquidity trap for *any* $a(0)_{-1}$ if the eigenvalues of $p_{00}F(0)$ are strictly inside the unit circle.[30] As it turns out, this condition on $p_{00}F(0)$ is a special case of the E-stability conditions applied to a model with ZLB regime persistence equal to p_{00} and an absorbing positive interest rate regime, which one can verify from the general E-stability conditions stated in Online Appendix B.3.[31] Therefore, E-stability is useful for predicting the possibility of diverging inflation dynamics under learning at the ZLB–even if agents' initial pessimism causes the ZLB to bind for an indefinite period of time. Figure 6 illustrates expectations-driven liquidity traps assuming the benchmark calibration and $p_{00} = 0.965$. As expected, PLT rules and AIT rules with long averaging window outperform the simple IT rule. Deflationary spirals occur under IT and AIT with short averaging window.

The condition on $p_{00}F(0)$ reveals that agents' *beliefs* about the persistence of the ZLB,

[30]Note that if $p_{00}F(0)$ has eigenvalues outside the unit circle then the economy can still escape the endogenous liquidity trap, but only for *some* initial conditions on $a(0)_{-1}$ (e.g., see Mertens and Ravn (2014)). Also note that agents' initial beliefs about $\Omega(s_t)$ introduce additional initial conditions that affect convergence (see Online Appendix B.4).

[31]From Online Appendix B.3., E-stability obtains if the real parts of the eigenvalues of $p_{00}F(0)$ and $F(1) = (I - A(1)\Omega(1))^{-1} A(1)$ are less than one. For our calibration, the eigenvalues of $F(1)$ are inside the unit circle.

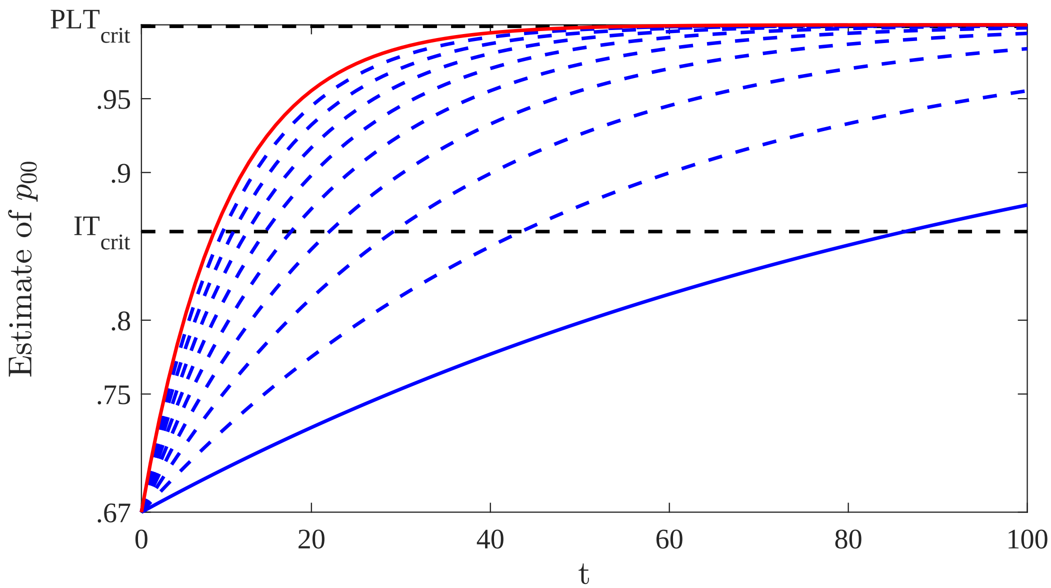
Figure 6: Learning in an Expectations-Driven Liquidity Trap



Note: The vertical axis represents the percentage deviation (e.g., 0.01 equals 1% quarterly inflation rate). Blue (red) lines depict E-stable (E-unstable) calibrations. In the E-stable simulations, the economy escapes the expectations-driven liquidity trap, whereas deflationary spirals occur in the E-unstable simulations.

i.e., p_{00}, matter for stability under learning at the ZLB, and not the actual duration of the ZLB events per se. This result is a consequence of a basic self-referential feature of the model: today's beliefs influence temporary equilibrium, which in turn, influences future beliefs. For instance, low inflation expectations are confirmed by low equilibrium inflation in the absence of active monetary policy, which can result in even lower future inflation expectations, and eventually, a deflationary spiral. However, this self-referential feature of the model is diminished by the expectation that the economy may escape the ZLB (i.e., $p_{00} < 1$). To see this, consider the baseline model (1)-(2) with IT (6), and note that $\hat{E}_t(x_{t+1}|s_t = 0) = p_{00}a(0)_{t-1} + (1 - p_{00})a(1)_{t-1}$ under the correctly-specified PLM (11), where $a(1)_{t-1}$ is fixed while $s_t = 0$. Only $a(0)_{t-1}$ evolves at the ZLB, and $\partial x_t/\partial a(0)_{t-1} = A(0)p_{00}$, so that the actual economy, x_t, is more responsive to changes in beliefs, $a(0)_{t-1}$, for higher values of p_{00}. Thus, we mitigate destabilizing feedback from expectations to reality and therefore to future expectations by decreasing the expected ZLB duration, p_{00}. This intuition also rationalizes why persistent (a high value of p_{11}) and frequent (a low value of p_{00}) active monetary regimes are key to the existence of E-stable REE, and suggests that a good monetary policy should promote determinacy and E-stability over the largest possible set of transition probabilities, (p_{00}, p_{11}). By this standard, the PLT rule (7) dominates IT

Figure 7: Learning the Transition Probability



Note: IT_{crit} and PLT_{crit} denote the maximum value of p_{00} that delivers an E-stable REE under IT and PLT, respectively. The solid red (blue) line shows learning dynamics when $\psi = 0.1$ ($\psi = 0.01$); dashed blue show dynamics for values of $\psi \in [0.01, 0.1]$.

(6) and AIT (8) in terms of stabilizing the economy.

5.3 Beliefs About the ZLB Duration

The analysis in this paper highlights that agents' beliefs about the persistence of the ZLB are an important determinant of E-stability, and as a starting point, we assumed fixed beliefs about p_{00} and p_{11}. In this section we deviate from our baseline model and consider two alternative assumptions about how agents forecast the end of ZLB events. The first alternative (in section 5.3.1) captures how adaptive learning agents might choose to learn the persistence of ZLB events. The second alternative (in 5.3.2) considers a ZLB case with a known exit date.

5.3.1 Learning the Persistence of the ZLB

Again, our earlier analysis assumed fixed beliefs about the regime transition probabilities, but agents could be expected to revise their transition probability beliefs, for example, if they experience a particularly deep, persistent liquidity trap. Therefore, we relax the assumption of the fixed transition probability belief, and let agents revise their beliefs about the transition probability using the following recursive estimator:

$$\hat{p}_{kk,t} = \hat{p}_{kk,t-1} + \psi(k)_{t-1}(\xi(k)_t - \hat{p}_{kk,t-1}) \tag{19}$$

where $\psi(k)_{t-1} > 0$ if $s_{t-1} = k$ and 0 otherwise, and $\xi(k)_t = 1$ if $s_t = k$ and 0 otherwise. In each period, agents also update their beliefs about the coefficients in their PLM (11), and time-t expectations are formed using recent estimates of the PLM coefficients and also $\hat{p}_{kk,t-1}$.[32] During a very prolonged liquidity trap, such as the episodes featured in Figure 6, agents' estimates of p_{00} could increase to levels where the E-stability conditions fail and inflation becomes dynamically unstable in real time. Figure 7 illustrates the evolution of beliefs about p_{00} according to (19) for different constant-gain parameters, $\psi(k_t) = \psi \in [0.01, 0.1]$ and given an initial belief equal to 0.67 in a ZLB event that lasts more than 100 consecutive quarters. The figure displays the maximum value of p_{00} that gives an E-stable REE under the IT and PLT rules, respectively, assuming the benchmark calibration and $p_{11} = 1$. Once the agents' estimate of p_{00} exceeds these maximum values, unstable dynamics may arise at the lower bound. It is evident that an IT rule poses risks to instability when agents learn about the transition probability. If they have a high gain parameter (e.g., $\psi = 0.1$), then unstable inflation dynamics may arise at the ZLB in fewer than 10 quarters. On the other hand, the PLT rule ensures stable inflation even for arbitrarily high values of $p_{00} < 1$, and hence evolving beliefs about the persistence of the ZLB will never threaten stability under PLT. Again, our results favor the PLT rule over its alternatives.

5.3.2 ZLB with a Known Exit Date

Earlier sections supposed a finite *expected* duration of ZLB events, which is given by $(1 - p_{00})^{-1}$, but the *actual* ZLB duration is stochastic. This section studies implications of a fully anticipated (maximum) actual and expected ZLB duration, equal to T. As before the model

[32] From the perceived transition probability law of motion, (19), $p_{kk,t}$ is entirely determined by s_t, and therefore we can study the evolution of perceived transition probabilities in isolation from the remaining equations in the model.

is given by (1)-(2) and one of the three policy rules above, and we assume $v_{s,t} = v_{d,t} = 0$ for brevity. The process for u_t is modified as follows:

$$u_t = \begin{cases} \bar{u} & \text{if } s_t = 0 \quad \text{for } t = 0, \ldots, T-1 \\ 0 & \text{if } s_t = 1 \quad \text{for } t = 0, \ldots, T-1 \\ 0 & \qquad\qquad\;\; \text{for } t \geq T \end{cases} \tag{20}$$

where $\bar{u} \leq 0$, and $p_{00} = Pr(s_{t+1} = 0 | s_t = 0) \leq 1$ for $0 \leq t < T-1$ and $p_{11} = Pr(s_{t+1} = 1 | s_t = 1) = 1$. Thus, agents wake up in a low demand state at time $t = 0$ and expect to remain in the low state in the following period with probability p_{00}. However, unlike before, the low demand state is only expected to last a maximum of T periods, after which time active monetary policy resumes (i.e., we tacitly assume $s_t = 1$ for $t \geq T$). In the special case of $p_{00} = 1$, agents expect the low demand state to last for exactly T periods, similar to the thought exercise explored in Cochrane (2017). Relatedly, the assumption $\bar{u} = 0$ and $p_{00} = 1$ replicates the calendar-based forward guidance exercises of Del Negro, Giannoni and Patterson (2015), among others. More generally, Eggertsson and Woodford (2003) take into account cases in which $p_{00} \leq 1$.[33]

As it turns out, we always have a unique mean-square stable and E-stable REE if agents believe the ZLB will be over by time T.

Proposition 3 *Consider (1)-(2) and suppose u_t follows (20) and T is finite. The model is determinate and the unique mean-square stable REE is E-stable under IT (6), PLT (7), or AIT (8) .*

Proof: See Appendix A.3. ■

The proof of the proposition builds on the solution approaches of Cagliarini and Kulish (2013), Kulish and Pagan (2017), and McClung (2021), and assumes that ϕ_π, ϕ_p and ϕ_y are sufficiently high to guarantee determinacy under any of the policy rules in the absence of

[33] Eggertsson and Woodford (2003) study a model that has stochastic ZLB duration, similar to the model introduced in Section 2. However, they also study optimal policy in a liquidity trap of stochastic duration under the assumption that the liquidity trap ends before some finite time T.

the ZLB regime ($s_t = 1$ always).[34] As in the case of stochastic duration, the *expectation* of exiting the ZLB in the future mitigates concerns about expectational instability. However, unlike the case of stochastic duration, the belief that the economy escapes the liquidity trap with certainty before some specific date ensures a unique mean-square stable equilibrium. Intuitively, the expectation that the ZLB can only last for a finite, specific number of periods implies a backward induction problem that coordinates the expectations and actions of rational agents: the expectation that the economy is in the locally unique REE near the steady state by $t = T$ determines $E_0 x_{T-1}$, which determines $E_0 x_{T-2}$ and therefore $\{E_0 x_t\}_{t=1}^{T-1}$ and the initial equilibrium response, x_0 (given x_{-1}).

E-stability follows from two facts about the framework. First, the MSV solution that agents attempt to learn can be expressed as a $T+1$-state solution of the form (10) where the first T states can only last for one period (see Appendix A.3. for details). Hence, agents cannot learn about the MSV coefficients for these first T states, which implies the absence of unstable learning dynamics while the ZLB is binding. Second, agents' PLM switches to a forecasting rule that is local to an E-stable REE of the linear model with active monetary policy once the economy escapes the ZLB (i.e., if $s_t = 1$ for $t < T$ or always for $t \geq T$). With stochastic duration considered in the previous sections, high values of p_{00} preclude E-stability because the economy can remain in the ZLB state *indefinitely* with some probability. However, in the framework implied by (20), the economy is guaranteed to transition into an absorbing, expectationally stable regime in finite time, and therefore the learning equilibrium may converge to the REE *asymptotically*.

Proposition 3 shows that the relative benefits of PLT vis-à-vis AIT or IT depends not only on the *expected* duration of ZLB events, but also on agents' beliefs about the *maximum* possible duration of ZLB events. PLT strictly dominates its alternatives with respect to our determinacy and E-stability criteria if agents are uncertain about how long a ZLB episode can last, but if agents are confident that the ZLB will end before a specific date, then all of the policy rules ensure an E-stable and unique mean-square stable REE. Overall, we find that PLT dominates its alternatives in terms of stabilizing expectations, but only weakly so

[34] Following Cagliarini and Kulish (2013), we can show that the unique mean-square stable REE is also the unique bounded equilibrium in the special case $p_{00} = 1$.

if agents expect the ZLB to end before a specific date.

6 Conclusion

This study evaluates policy rules that respond to average inflation and price level as well as output instead of the period inflation rate (a standard Taylor rule) using the criteria of determinacy of an REE and the learnability of the equilibrium in a standard New Keynesian model subject to persistent, recurring ZLB episodes. Our results are strongly in favor of the price level targeting framework as effective stabilization policy, which gives a unique, learnable equilibrium in models even with extremely persistent ZLB events. Thus, under price level targeting, policymakers should be less worried about sunspots, and deflationary spirals under learning. Nominal GDP targeting can be understood as a special case of the price level targeting rules when the reaction coefficients on price level and output are the same. We also find that average inflation targeting rules can promote determinacy and E-stability very effectively, provided that the measure of average inflation is sufficiently backward looking. However, standard Taylor rules that implement a simple inflation targeting policy are prone to indeterminacy and possibly E-instability, unless agents haved fixed beliefs about the maximum duration of a ZLB event. These findings have important implications for stabilization policy in the current low interest rate environment.

Several avenues for future work are available. For instance, we take the frequency of ZLB events as given in order to show that the expected duration and frequency of these events are key determinants of determinacy and E-stability under various policy rules. Future work might instead investigate whether some policy rules help to avoid liquidity traps altogether, or minimize their welfare costs when they do occur under learning. Future work should also address E-stability under the infinite-horizon learning approach of Preston (2005), and also assess the uniqueness of *bounded* equilibrium, following the methodology of Barthélemy and Marx (2017) and Barthélemy and Marx (2019). Barthélemy and Marx (2017) in particular provide a perturbation approach which can be applied to the underlying non-linear model, and which allows for endogenous transition probabilities between policy regimes. We leave the important issue of determinacy in the bounded stability sense for future research.

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Appendices

A.1. Proof of Proposition 1

Consider a slightly modified version of the Taylor rule (6):

$$i_t = \tilde{s}_t(\phi_\pi \pi_t + \phi_y y_t) - (1 - \tilde{s}_t)\bar{i}$$

where $\tilde{s}_t = \epsilon$ if $s_t = 0$ and ϵ is some arbitrarily small positive constant; and $\tilde{s}_t = 1$ otherwise. We introduce $\tilde{s}_t$ to ensure that the inflation process is well defined when we combine the modified Taylor rule with (12), yielding the following Markov-switching expectational difference equation for inflation:

$$\pi_t = (\phi_\pi \tilde{s}_t)^{-1} E_t \pi_{t+1} + (\phi_\pi \tilde{s}_t)^{-1} \left((1 - \tilde{s}_t)\,\bar{i} + \sigma u_t\right) \tag{21}$$

where $\phi_\pi > 0$ and $y_t = 0$ for all t is imposed in (6) with flexible prices.

Assume $\phi_\pi \geq 0$ and $p_{00} + p_{11} > 1$. From Cho (2021),[35] (21) is determinate if and only if

$$r(F) = r \begin{pmatrix} p_{11}(\phi_\pi)^{-2} & p_{10}(\phi_\pi)^{-2} \\ p_{01}(\epsilon)^{-2} & p_{00}(\epsilon)^{-2} \end{pmatrix} < 1$$

where $p_{10} = 1 - p_{11}$, $p_{01} = 1 - p_{00}$, and $r(F)$ denotes the spectral radius of the matrix F.

[35] See Appendix B.2. for further details.

The eigenvalues of F, λ_1 and λ_2, are the roots of the following quadratic equation:

$$f(\lambda) = \lambda^2 - (p_{00}(\epsilon)^{-2} + p_{11}(\phi_\pi)^{-2})\lambda + (p_{11} + p_{00} - 1)\phi_\pi^{-2}\epsilon^{-2} = 0$$

As demonstrated on p. 28 of LaSalle (1986), both eigenvalues, λ_1 and λ_2, are inside the unit circle if and only if

$$\begin{aligned} |(p_{11} + p_{00} - 1)\phi_\pi^{-2}\epsilon^{-2}| &< 1 \\ |p_{00}\epsilon^{-2} + p_{11}\phi_\pi^{-2}| &< 1 + (p_{11} + p_{00} - 1)\phi_\pi^{-2}\epsilon^{-2}. \end{aligned}$$

The first condition for determinacy, $|(p_{11}+p_{00}-1)\phi_\pi^{-2}\epsilon^{-2}| < 1$, is surely violated for $\phi_\pi < \infty$ as $\epsilon \to 0$. Hence, the model (21) is indeterminate when $\epsilon \approx 0$.

From McClung (2020), we obtain E-stability of the MSV solution to (21) if

$$r^e(A) = r^e \begin{pmatrix} p_{11}(\phi_\pi)^{-1} - 1 & p_{10}(\phi_\pi)^{-1} \\ p_{01}(\epsilon)^{-1} & p_{00}(\epsilon)^{-1} - 1 \end{pmatrix} < 0,$$

where $r^e(A)$ denotes the maximum of the real parts of the eigenvalues of A. Because the trace of A, $tr(A) = p_{00}(\epsilon)^{-1} + p_{11}(\phi_\pi)^{-1} - 2 > 0$ for small ϵ, at least one eigenvalue of A is positive as ϵ approaches zero. Hence, the MSV solution is E-unstable.

A.2. Proof of Proposition 2

Consider (14) and assume $\phi_p \geq 0$ and $p_{00} + p_{11} > 1$. Then, (14) is determinate and the unique REE is E-stable if and only if $\phi_p > 0$ and $p_{00} < 1$. From Cho (2021),[36] (14) is

[36] See A.3 for further details.

determinate if and only if

$$r(F) = r\begin{pmatrix} p_{11}(1+\phi_p)^{-2} & p_{10}(1+\phi_p)^{-2} \\ p_{01} & p_{00} \end{pmatrix} < 1$$

where $p_{10} = 1 - p_{11}$, $p_{01} = 1 - p_{00}$, and $r(F)$ denotes the spectral radius of the matrix F. The eigenvalues of F, λ_1 and λ_2, are the roots of the following quadratic equation:

$$f(\lambda) = \lambda^2 - (p_{00} + p_{11}(1+\phi_p)^{-2})\lambda + (p_{11} + p_{00} - 1)(1+\phi_p)^{-2} = 0$$

As demonstrated on p. 28 of LaSalle (1986), both eigenvalues, λ_1 and λ_2, are inside the unit circle if and only if

$$\begin{aligned} |(p_{11} + p_{00} - 1)(1+\phi_p)^{-2}| &< 1 \\ |p_{00} + p_{11}(1+\phi_p)^{-2}| &< 1 + (p_{11} + p_{00} - 1)(1+\phi_p)^{-2}, \end{aligned}$$

which holds provided that $p_{00} + p_{11} - 1 > 0$, $\phi_p > 0$, and $p_{00} < 1$. From McClung (2020), E-stability of the MSV solution to (14) is obtained if

$$r^e(A) = r^e\begin{pmatrix} p_{11}(1+\phi_p)^{-1} - 1 & p_{10}(1+\phi_p)^{-1} \\ p_{01} & p_{00} - 1 \end{pmatrix} < 0$$

where $r^e(A)$ denotes the maximum of the real parts of the eigenvalues of A. Because the trace of A is negative (i.e., $tr(A) = p_{11}(1+\phi_p)^{-1} + p_{00} - 2 < 0$), and the determinant of A is positive (i.e., $det(A) = (1 - p_{00})(1 - 1/(1+\phi_p)) > 0$) under the assumptions in Proposition 2, both eigenvalues of A have negative real parts. Thus, we have E-stability of the MSV solution to (14).

A.3. Proof of Proposition 3

First, obtain the model solution for $t \geq T$. For $t \geq T$, the model assumes the form:

$$x_t = A^* E_t x_{t+1} + B^* x_{t-1} \tag{22}$$

where $(A^*, B^*) = (A(1), B(1))$ and $(A(1), B(1))$ are defined in the main text (i.e., see (9)).[37] If ϕ_π and ϕ_y are sufficiently large (e.g., $\phi_\pi > 1$ under IT or $\phi_p > 0$ under PLT), then the unique REE for $t \geq T$ assumes the form

$$\begin{aligned} x_t &= \Omega^* x_{t-1} \\ \Omega^* &= \left(I - A^* \Omega^*\right)^{-1} B^* \end{aligned} \tag{23}$$

One can easily show that (23) is the unique REE law of motion for x_t, $t \geq T$ using a variety of standard linear rational expectation techniques.[38] For $t = 0, \ldots, T-1$ the model is given by

$$x_t = \begin{cases} A^* E_t x_{t+1} + B^* x_{t-1} & \text{if } s_t = 1 \\ A(0) E_t x_{t+1} + B(0) x_{t-1} + C(0) & \text{if } s_t = 0 \end{cases}$$

[37] Note that $C^* = C(1) = 0$ in this case. Also recall that we set $v_t = 0$ for simplicity, which allows us to write $D^* = D(0) = D(1) = 0$, but results are not sensitive to this assumption.

[38] See, e.g., Blanchard and Kahn (1980). Standard linear RE techniques apply to this model class because of its linear form for $t \geq T$. See Cagliarini and Kulish (2013), Kulish and Pagan (2017) and Gibbs and McClung (2022) for more.

where $A(0), B(0), C(0)$ are also defined in the main text. Let $x_t^j = x_t | s_t = j$ for $j = 0, 1$. It is immediately apparent that $x_t^1 = \Omega^* x_{t-1}$ for all t. For $t = T-1$ we have

$$\begin{aligned} x_{T-1}^0 &= A(0)E_{T-1}(x_T | s_{T-1} = 0) + B(0)x_{T-2}^0 + C(0) \\ &= A(0)\Omega^* x_{T-1}^0 + B(0)x_{T-2}^0 + C(0) \\ &= (I - A(0)\Omega^*)^{-1} B(0)x_{T-2}^0 + (I - A(0)\Omega^*)^{-1} C(0) \\ &= \Omega(0)_{T-1} x_{T-2}^0 + \Gamma(0)_{T-1} \end{aligned}$$

Therefore we have

$$\begin{aligned} x_{T-2}^0 &= (I - A(0)(p_{00}\Omega(0)_{T-1} + (1-p_{00})\Omega^*))^{-1} B(0) x_{T-3}^0 \\ &+ (I - A(0)(p_{00}\Omega(0)_{T-1} + (1-p_{00})\Omega^*))^{-1}(A(0)p_{00}\Gamma(0)_{T-1} + C(0)) \\ &= \Omega(0)_{T-2} x_{T-3}^0 + \Gamma(0)_{T-2} \end{aligned}$$

Proceeding recursively backward in time:

$$\begin{aligned} \Omega(0)_t &= (I - A(0)(p_{00}\Omega(0)_{t+1} + (1-p_{00})\Omega^*))^{-1} B(0) \\ \Gamma(0)_t &= (I - A(0)(p_{00}\Omega(0)_{t+1} + (1-p_{00})\Omega^*))^{-1} (A(0)p_{00}\Gamma(0)_{t+1} + C(0)) \end{aligned}$$

from $t = T-2$ to $t = 0$. The solution for $t \geq 0$ is therefore given by:

$$x_t = \begin{cases} \Omega(0)_t x_{t-1} + \Gamma(0)_t & \text{if } \ s_t = 0 \\ \Omega^* x_{t-1} & \text{if } \ s_t = 1 \ \text{ or } \ t \geq T. \end{cases} \tag{24}$$

Define $F(0)_t$ as follows:

$$F(0)_t = (I - A(0)(p_{00}\Omega(0)_{t+1} + (1-p_{00})\Omega^*))^{-1} A(0)$$

for $t = 0, \ldots, T-2$, and

$$F(0)_{T-1} \quad = \quad (I - A(0)\Omega^*))^{-1}A(0)$$

Next, we recast the model and model solution in the form (9) and (10), respectively, and assess mean-square stability, E-stability, and uniqueness of the REE (24). Let $\tilde{s}_t \in \{0, 1, \ldots, T\}$ denote a $T+1$-state Markov process with transition matrix:

$$\tilde{P} = \begin{pmatrix} 0 & p_{00} & 0 & \ldots & 0 & 1-p_{00} \\ 0 & 0 & p_{00} & \ldots & 0 & 1-p_{00} \\ \vdots & & & \ddots & & \vdots \\ 0 & \ldots & & & p_{00} & 1-p_{00} \\ 0 & \ldots & & & 0 & 1 \\ 0 & \ldots & & & 0 & 1 \end{pmatrix}$$

Let $\tilde{s}_t = t$ if $0 \leq t < T$ *and* $s_t = 0$; otherwise, $\tilde{s}_t = T$.[39] This implies restrictions on the (i,j)-element of $\tilde{P}$, $\tilde{P}_{ij}$: $\tilde{P}_{ij} = Pr(\tilde{s}_{t+1} = j | \tilde{s}_t = i) = p_{00}$ if $j \leq T-1$ and $i = j-1 \geq 0$; $\tilde{P}_{iT} = 1 - p_{00}$ for $0 \leq i < T-1$ and $\tilde{P}_{iT} = 1$ for $i \geq T-1$; otherwise, $\tilde{P}_{ij} = 0$. Similarly, let $(A(\tilde{s}_t), B(\tilde{s}_t), C(\tilde{s}_t)) = (A(0), B(0), C(0))$, $\tilde{\Omega}(\tilde{s}_t) = \Omega(0)_{\tilde{s}_t}$, $\tilde{\Gamma}(\tilde{s}_t) = \Gamma(0)_{\tilde{s}_t}$ and $\tilde{F}(\tilde{s}_t) = F(0)_{\tilde{s}_t}$ for $\tilde{s}_t = 0, \ldots, T-1$ and $(A(T), B(T), C(T)) = (A^*, B^*, 0)$, $\tilde{\Omega}(T) = \Omega^*$, $\tilde{\Gamma}(T) = 0$ and $\tilde{F}(T) = F^* = (I - A^*\Omega^*)^{-1}A^*$. The model and solution are now in the form (9) and (10), respectively. Following Cho (2021) (see also Online Appendix B.2.), the solution (24) is

[39] Note that to recover the specific model under consideration we impose the restriction: $\tilde{s}_0 \in \{0, T\}$. However, the proof of Proposition 3 applies to the more general case: $\tilde{s}_0 \in \{0, 1, \ldots, T\}$.

mean-square stable if and only if

$$
\begin{aligned}
& r\begin{pmatrix}
0 & 0 & 0 & \dots & 0 & 0 \\
p_{00}\tilde{\Omega}(1)\otimes\tilde{\Omega}(1) & 0 & 0 & \dots & 0 & 0 \\
0 & p_{00}\tilde{\Omega}(2)\otimes\tilde{\Omega}(2) & 0 & \dots & 0 & 0 \\
\vdots & & & \ddots & & \vdots \\
0 & \dots & & & 0 & 0 \\
(1-p_{00})\Omega^*\otimes\Omega^* & (1-p_{00})\Omega^*\otimes\Omega^* & \dots & & \Omega^*\otimes\Omega^* & \Omega^*\otimes\Omega^*
\end{pmatrix} \\
= \; & r\left(\Omega^*\otimes\Omega^*\right) < 1
\end{aligned}
$$

where $r(A)$ denotes the spectral radius of A and $r(\Omega^*\otimes\Omega^*) < 1$ is implied by the fact that (23) is the unique linear REE for the model structure for $t \geq T$. This proves that the REE is mean-square stable.

Next, consider uniqueness. From Cho (2021), the model admits a unique mean-square stable REE if and only if

$$
\begin{aligned}
& r\begin{pmatrix}
0 & p_{00}\tilde{F}(0)\otimes\tilde{F}(0) & 0 & \dots & 0 & (1-p_{00})\tilde{F}(0)\otimes\tilde{F}(0) \\
0 & 0 & p_{00}\tilde{F}(1)\otimes\tilde{F}(1) & \dots & 0 & (1-p_{00})\tilde{F}(1)\otimes\tilde{F}(1) \\
\vdots & & & \ddots & & \vdots \\
0 & \dots & & & 0 & \tilde{F}(T-1)\otimes\tilde{F}(T-1) \\
0 & \dots & & & 0 & F^*\otimes F^*
\end{pmatrix} \\
= \; & r\left(F^*\otimes F^*\right) < 1
\end{aligned}
$$

where $r(F^*\otimes F^*) < 1$ again follows from the fact that (23) is the unique REE law of motion for x_t for $t \geq T$ (e.g., see Cho (2021) or Appendix B.2). Hence, (24) is the unique mean-square stable REE of the model. E-stability follows from Propositions 1 and 2 of McClung (2020).